Investment project valuation : A new equity perspective

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ABSTRACT

We suggest a new approach to calculating a project's net present value, termed the "displaced equity method". Based on a straightforward formula, it analyzes a project partially financed with debt from the perspective that every year the amount of outstanding debt displaces an equivalent amount of equity that otherwise would be tied up in the project. Although they represent distinct shareholders' perspectives, the displaced equity method and the equity residual method lead to the same investment decision. Every year, the project's value calculated with the displaced equity method is equal to the sum of the project's debt and equity values. In practice, when the schedule of expected outstanding debt amounts is known, using the displaced equity method is an easy way to estimate the project's net present value.

INTRODUCTION

To value a project, the key issue is to know if the firm undertaking this project targets a constant debt-to-value ratio. If such is the case, the project's value is obtained by discounting the project's operating cash flows at the firm's constant WACC discount rate (computed with the firm's target debt ratio). To determine the project's equity value by using the equity residual method, one must then consider a (theoretical) debt repayment schedule such that every year the project's debt ratio is equal to the firm's target debt ratio. In practice, the equity residual method is used when a predetermined stream of debt cash flows can be associated[1] with the project under study. The project's equity value can then be calculated by considering this stream of debt cash flows. In this short note, we stick to the operational viewpoint usually adopted by industrial practitioners who define the project's cost of equity as being the minimum return-on-equity value required by the firm to undertake the project (under the assumptions made about financing). In other words, we consider that the project's cost of equity remains unchanged[2] over the project's lifetime.

In this context, we suggest a new equity perspective that leads to a novel and straightforward valuation formula that does not require equity residual cash flows to be computed. Every year, it values an investment project partially financed with debt as the sum of its debt and equity values. This perspective is therefore that of shareholders considering the full project's economic value, whereas, in the usual equity residual method, shareholders only consider their invested equity. To determine the project's economic value has a practical interest: it represents the minimum cash amount at which the firm is willing to sell its stake in the project at any given point in time.

The paper is structured as follows. In the next section, notations are adopted and the usual equity perspective - i.e., the equity residual method - is briefly recalled. Next, the new equity perspective –i.e., the displaced equity method - is introduced. We prove that the usual and new equity perspectives are consistent. The practical interest of the displaced equity method is emphasized in the last section.

[1] This may happen in various instances: the project is the sole project of the firm (at least in a given class of risk), or it is financed by project finance, or the firm's stake in the project is consolidated by using the equity method (as defined in IAS 28), i.e. no debt associated with the project appears in the firm's consolidated balance sheet.

[2] The financial literature suggests formulas that yield a project's cost of equity susceptible to change over time when dollar amounts of outstanding debt are targeted; for instance, Inselbag and Kaufold (1997) derive a formula consistent with the adjusted-present-value approach, Ruback (2002) produces a formula consistent with the CAPM approach. When a fixed debt ratio is targeted, the cost of equity remains constant (see for instance Miles and Ezzell (1985), Harris and Pringle (1985) and Taggart (1991)). We further discuss this point in the last section.

THE USUAL EQUITY PERSPECTIVE: THE EQUITY RESIDUAL METHOD

Let us consider a project generating every year t the after-tax operating cash flow[3] f_t. The project's lifetime extends from year 0 to T (i.e., $f_t = 0$ when $t > T$). The project is partially financed with debt. D_t denotes the amount of outstanding debt associated with the project at the end of year t (with $D_t = 0$ when $t \geq T$, and $D_{-1} = 0$). This debt is contracted at the annual interest rate k_d. The interest payments are assumed to generate tax shields at the corporate tax rate τ. The cost of equity is denoted as k_e.

As recalled by Chambers *et al.* (1982), the equity residual method - also called flow-to-equity method - values the project (or the firm) under study from the invested-equity perspective. The cash flows considered are those available for distribution to the shareholders (i.e., each year, the after-tax operating cash flow minus the principal repayment and the after-tax interest payment). Each year t, the market value of equity E_t is the sum of the present values of subsequent equity residual cash flows:

$$E_t = \sum_{s=t+1}^{T} \frac{f_s + D_s - \left(1 + \left(1-\tau\right) k_d\right) D_{s-1}}{\left(1+k_e\right)^{s-t}} \qquad (1)$$

(1) is equivalent to:

$$E_{t-1} = \frac{E_t + D_t - \left(1 + \left(1-\tau\right) k_d\right) D_{t-1} + f_t}{1+k_e} \qquad (2)$$

with $E_T = 0$.

THE NEW EQUITY PERSPECTIVE: THE DISPLACED EQUITY METHOD

We now adopt the point of view of shareholders who consider the full project's value, and not their invested equity only. In this perspective, the debt amount associated with the project every year is considered as just replacing an equivalent amount of equity which, otherwise, would be tied-up in the project. The reinvestment of this displaced equity by the shareholders yields a return equal to their cost of equity[4] k_e.

In other words, in any year $t-1$, the oustanding amount of debt D_{t-1} is merely viewed as a means of "freeing" an equivalent amount of equity that is immediately invested by the shareholders for one year at the return k_e. The cash flow yielded by this investment in the following year is $\left(1+k_e\right) D_{t-1}$. The actual gain for the shareholders in year t is given by the

[3] Here, this term refers to the cash flow of the project before any financial claims are paid; for tax purposes, the taxable income used is defined as the earnings before interest and taxes, which means that the after-tax operating cash flow includes no interest tax shields; some authors also use the term "free cash flow".

[4] The cost of equity is here viewed as an opportunity cost, as the displaced amount of equity is implicitly assumed to be invested in other assets with a similar risk for the shareholders; this equity displacement remains a mere perspective, in the sense that the cost-of-equity value considered here is that corresponding to the financial structure of the project under study.

difference between this cash flow and the sum of the after-tax interest payment and debt repayment:

$$\left(1+k_e\right)D_{t-1}-\left(1+\left(1-\tau\right)k_d\right)D_{t-1}=\left(k_e-\left(1-\tau\right)k_d\right)D_{t-1}$$

Therefore, in the displaced equity method, the shareholders' cash flow in year *t* is:

$$f_t+\left(k_e-\left(1-\tau\right)k_d\right)D_{t-1}$$

The project's value V_t in year *t* is consequenty:

$$V_t=\sum_{s=t+1}^{T}\frac{f_s+\left(k_e-\left(1-\tau\right)k_d\right)D_{s-1}}{\left(1+k_e\right)^{s-t}}$$

Or, equivalently:

$$V_{t-1}=\frac{f_t+V_t+\left(k_e-\left(1-\tau\right)k_d\right)D_{t-1}}{1+k_e} \tag{3}$$

In particular, in year 0, by using the notation $D_{-1}=0$, the project's net present value computed with the displaced equity method is:

$$\sum_{s=0}^{T}\frac{f_s+\left(k_e-\left(1-\tau\right)k_d\right)D_{s-1}}{\left(1+k_e\right)^{s}}$$

CONSISTENCY OF THE TWO EQUITY PERSPECTIVES

The following propositions are verified:

(i) each year *t*, the project's value obtained with the displaced equity method is equal to the sum of the project's debt and equity values:

$$V_t=D_t+E_t \quad \left(\forall t\right) \tag{4}$$

(ii) the equity residual method and the displaced equity method yield the same net present value and internal rate of return[5].

Both equity perspectives are therefore consistent, since they lead to the same investment decision, as stated by (ii). However, as shown by (i), the displaced equity method leads to compute the "total" project's value (and not only the equity value).

Proof of (i) and (ii)

(i) is proved by recurrence:

In year T, (4) obviously holds since we have:

$$V_T=E_T+D_T=0$$

Let us now assume that (4) holds in year *t*: $V_t=E_t+D_t$. By subtracting (2) from (3) we obtain:

[5] Note that the internal rate of return *r* yielded by the displaced equity method is given by the following formula:

$$\sum_{s=0}^{T}\frac{f_s+\left(r-\left(1-\tau\right)k_d\right)D_{s-1}}{\left(1+r\right)^{s}}=0$$

$$V_{t-1} - E_{t-1} = \frac{V_t - E_t - D_t + (1+k_e) D_{t-1}}{1+k_e} \quad (5)$$

Since we assume that $V_t - E_t - D_t = 0$, (5) gives:

$$V_{t-1} - E_{t-1} = D_{t-1}$$

The recurrence is over: (4) holds in any year *t*.

In year 0, (4) gives:

$$V_0 + f_0 = E_0 + D_0 + f_0 \quad (6)$$

The left-hand side of (6) is the net present value computed with the displaced equity perpsective and the right-hand side of (6) is the net present value obtained with the equity residual method. Both net present values are therefore equal.
Since both equity perspectives use the same cost-of-equity discount rate and yield the same net present value, they necessarily yield the same internal rate of return. (ii) is therefore proved.

PRACTICAL INTEREST OF THE DISPLACED EQUITY METHOD

To value a given project, the displaced equity method just requires to know the schedule of the amounts of outstanding debt associated with this project. Every year, the displaced-equity cash flow is the project's after-tax operating cash flow adjusted in a very simple way. Clearly, when the expected outstanding debt amounts are known, using the displaced equity method is the easiest way to calculate the project's net present value, since it does not require the equity residual cash flows to be explicitly computed. For this reason, this new method may be of practical interest for those who are in charge of project valuation in industry or financial institutions.
As mentioned in the introduction, under certain financing assumptions the financial literature recommends using a cost-of-equity value that varies through time. In theory, the project's value must then be computed according to a rather delicate and complex process. In this case, the equity residual method, and therefore the displaced equity method, yield projects' values that are equal to those obtained with other methods (see for instance Inselbag and Kaufold (1997)), provided that the cost-of-equity rates successively used are correctly estimated. We could have introduced the displaced equity method by considering a cost-of-equity rate varying over the project's lifetime, but doing this would have rendered this method – as well as the equity residual method – non-operational whereas calculating a project's equity value and return on equity is widespread practice. For practitioners, assuming a constant cost of equity and using the displaced equity method should most probably be an easy and pragmatic way of estimating a project's value.

REFERENCES

Chambers, D.R., Harris, R.S. and Pringle, J.J. (1982) Treatment of financing mix in analyzing investment opportunities. *Financial Management*, 8, 24-41.
Harris, R.S. and Pringle, J.J. (1985) Risk-adjusted discount rates – extensions from the average-risk case. *Journal of financial research*, 8, 237-244.
Inselbag, I. and Kaufold, H. (1997) Two DCF approaches for valuing companies under alternative financing strategies (and how to choose between them). *Journal of Applied Corporate Finance*, 10, 114-122.

Miles, J. and Ezzell, J.R. (1985) Reformulating tax shield valuation: a note. *Journal of finance*, XL, 1485-1492.
Ruback, R.S. (2002) Capital cash flows: a simple approach to valuing risky cash flows. *Financial Management*, 31, 85-103.
Taggart, R.A. (1991) Consistent valuation and cost of capital expressions with corporate and personal taxes. *Financial Management*, 20, 8-20.

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