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► **To cite this version:**

Grégory Rousseau, Delphine Sinoquet, Antonio Sciarretta, Yohan Milhau. Design Optimization and Optimal Control for Hybrid Vehicles. EngOpt 2008 -International Conference on Engineering Optimization, Jun 2008, Rio de Janeiro, Brazil. hal-02284311

HAL Id: hal-02284311

<https://ifp.hal.science/hal-02284311>

Submitted on 11 Sep 2019

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Design Optimization and Optimal Control for Hybrid Vehicles

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1. Abstract

Growing environmental and global crude oil supplies concerns are stimulating research on new vehicle technologies. Hybrid-electric vehicles appear to be one of the most promising technologies for reducing fuel consumption and pollutant emissions. Different types of hybrid-electric powertrains exist: from the mild-hybrid vehicle, equipped with a small electric motor, to the combined hybrid like the Toyota prius.

This paper presents a parametric study focused on variations of the size of the powertrain components, and optimization of the power split between the engine and electric motor with respect to fuel consumption.

To perform this optimization on a prescribed driving cycle (for instance, the New European Driving Cycle), a dynamic programming algorithm based on a reduced model is implemented. This simplified model allows a fast optimization with a fine parameterization of the controller: it furnishes the optimal power repartition at each time step regarding fuel consumption under constraints on the battery state of charge. The obtained results may be used to determine the best components of a given powertrain, for a prescribed vehicle cycle.

The optimal split obtained thanks to dynamic programming algorithm can not be used directly on a vehicle as a real time control law, as the future can not be known in advance in normal driving conditions. To overcome this difficulty, we implement, as a real-time strategy, the Equivalent Consumption Minimization Strategy (ECMS): the battery being considered as an auxiliary reversible fuel reservoir, an instantaneous minimization of ECMS is performed. This control law is inferred from Pontryagin's Minimum Principle, where the Lagrange multiplier can be deduced from previous optimization results on given driving cycles. Offline optimization results and real-time control laws are compared for a realistic hybrid vehicle application.

2. Keywords: Hybrid vehicles, optimal control, dynamic programming, Pontryagin's principle.

3. Introduction

Growing environmental concerns coupled with concerns about global crude oil supplies stimulate research on new vehicle technologies. Hybrid-electric vehicles (HEV) appear to be one of the most promising technologies for reducing fuel consumption and pollutant emissions [1]. Indeed, it is well known that the kinetic energy gained by the classical vehicle after an acceleration can not be recuperated when braking. One of the advantage of the hybrid vehicle is its ability to recuperate a part (depending of efficiencies) of this energy, to store it in a battery, and to re-use it latter. This electric energy can therefore be used by an electric motor to power the vehicle and to save fuel.

The control of hybrid powertrains is more complex than control of classic engine. Actually, the control laws have to deal with the state of charge of the battery, which provides the level of remaining electric energy, and with the variable efficiency of each element of the powertrain. Optimization of energy management strategies on given driving cycles is often used to derive sub-optimal control laws to be implemented on the vehicle (see among others [2], [3], [4], [5]).

In this paper, we focus on a parallel hybrid powertrain. The vehicle used for this parametric study is a Citroën C1, with a gasoline engine capacity of 1000cc. From this vehicle, we consider an additional source of power given by an electric motor located near the wheels. This motor takes its energy from an additional battery. A model of this powertrain has been implemented in AMESim* environment for validation purposes, and also in Matlab, for optimization. The latter model is a simplified static model, that allows fast computations. Two different optimization algorithms are presented and applied on this model, depending on the needs:

*AMESim is a LMS software to build system models with physics-based building blocks

- a classical Dynamic Programming algorithm ([2], [3], [4]), which provides the optimal control strategy on a prescribed cycle, and the potential of a powertrain.
- a sub-optimal control law, based on the Principle of Pontryagin, used as a real-time control law.

4. System Modelling and Optimal Control Problem

4.1. Optimal Control Problem

The optimal control problem under study consists in minimizing the fuel consumption of the vehicle along a prescribed vehicle cycle, taking into account physical constraints from battery, engine and electric motor. The general optimization problem is the following:

$$\begin{cases} \min_u \left\{ J(u) := \int_0^T L(x(t), u(t), t) dt + g(x(T), T) \right\} \\ \text{subject to:} \\ \dot{x}(t) = f(x(t), u(t), t), \quad x(0) = x_0 \\ x_{min} \leq x(t) \leq x_{max} \\ u_{min}(t) \leq u(t) \leq u_{max}(t) \end{cases} \quad (1)$$

where 0 and T are respectively the initial and the final times of the prescribed driving cycle, $u(t)$ is the control, $x(t)$ is the state, $L(x(t), u(t), t)$ is the instantaneous fuel consumption, $f(x(t), u(t), t)$ is the function that controls the variations of the dynamic system, and $g(x(T), T)$ is a penalization term handling a constraint on the final state of charge of the battery.

In this problem, the state has two components:

$$x(t) = \begin{cases} x_1(t) : & \text{the state of charge (SOC) of the battery} \\ x_2(t) : & \text{the state of the engine (on/off)}. \end{cases} \quad (2)$$

While $x_1(t)$ is a continuous state governed by $\dot{x}_1(t) = f_1(u(t), t)$, the second state $x_2(t)$ takes only the values 0 or 1 corresponding to *engine stop* / *engine start*. The introduction of $x_2(t)$ permits to take into account an extra fuel consumption when the engine is being started.

The control vector has three components, to be optimized:

$$u = \begin{cases} u_1 : & \text{the power split ratio between the engine and the electric motor} \\ u_2 : & \text{the gear} \\ u_3 : & \text{starting up / stopping the engine} \end{cases} \quad (3)$$

where u_1 is the classical control for HEV, u_2 may be optimized too, especially for automatic transmissions or CVT transmissions (continuous variation of the gear ratio), and u_3 is linked to x_2 state.

In this optimization problem, the fuel consumption is given by

$$L(x(t), u(t), t) = \begin{cases} L_1(u_1(t), t) & \text{if } x_2 = 1 & \text{(engine started)} \\ 0 & \text{if } x_2 = 0 & \text{(engine stopped)} \\ L_2 & \text{if } x_2 = 0 \text{ and } u_3 = 1 & \text{(engine start-up)} \end{cases} \quad (4)$$

where $L_1(u_1(t), t)$ is the fuel consumption given by a map depending on engine speed $\omega_e(t)$ and provided torque $T_e(t)$, and L_2 is the extra fuel consumption at engine start-up.

This minimization problem must be solved on a prescribed cycle, for instance the NEDC cycle, Figure 1. Thanks to a vehicle model (mass, wheel diameter, aerodynamic coefficients, etc.) and the speed request $V(t)$, the requested torque and speed at the wheels are computed: $(T_w(t), \omega_w(t))$. In this optimization problem, we consider the cycle as a wheel torque $T_w(t)$ and wheel speed $\omega_w(t)$ trajectory to be followed.

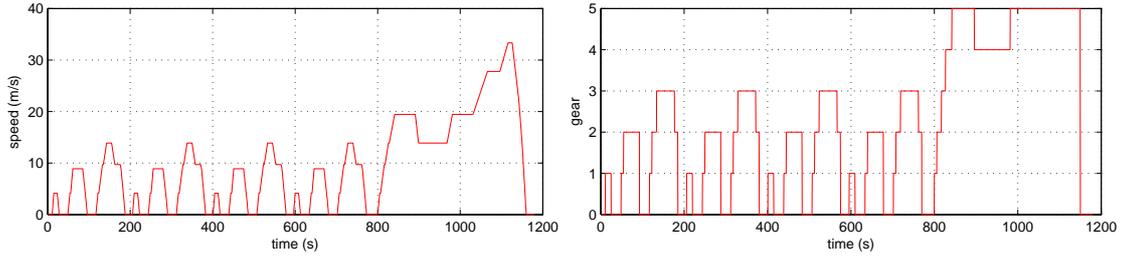


Figure 1: NEDC driving cycle: vehicle speed at left / gear ratio at right

In this application, we consider that the engine is located upstream of the gearbox, as on conventional vehicles. Besides, the electric motor is located downstream of the gearbox, beside the wheels. Let us introduce now some useful notations to describe the powertrain and to define the control $u(t)$:

- the requested torque at wheels is written as $T_w(t) = R_{far}R_{gr}T_e(t) + T_m(t) + T_{bk}(t)$, and can be achieved thanks to the engine torque $T_e(t)$ multiplied by the front axle ratio R_{far} and by the gear ratio R_{gr} , and thanks to the motor torque $T_m(t)$. A brake torque $T_{bk}(t)$ can also be applied by braking system.
- the engine torque is $T_e(t) = u(t)T_w(t)/(R_{far}R_{gr})$,
- the electric motor torque is $T_m(t) = (1 - u(t))T_w(t)$.

During braking phases, $T_{rq}(t) < 0$, the braking torque can complete the electric motor torque to slow down the vehicle: $T_{bk}(t) = T_w(t) - T_m^{min}(t)$ with $T_m^{min}(t)$ the minimal electric motor torque depending on the engine speed, thus depending on time.

A simple model of the battery is implemented, the state of charge of the battery evolving as

$$\dot{x}_1(t) = -\frac{\omega_m(t)T_m(t)K'}{U_{batt}(t)n_{capa}} \quad (5)$$

with $\omega_m(t)$ the electric motor speed, $U_{batt}(t)$ the battery voltage, K' a scaling constants, and n_{capa} the nominal capacity of the battery. In the following, we assume that the battery voltage is constant along the cycle. We set $K = \frac{K'}{U_{batt}.n_{capa}}$ and by replacing $T_m(t)$ by its expression, we obtain

$$\dot{x}_1(t) = -K\omega_m(t)(1 - u(t))T_w(t). \quad (6)$$

In this optimal control problem, we make several assumptions about the powertrain: (i) the wheel speed ω_w is equal to the electric motor speed ω_m , (ii) as the optimization problem is applied on a full hybrid vehicle, the engine can be stopped when the torque is provided only by the electric motor. As mentioned above in the definition of $L(x(t), u(t), t)$, a fuel consumption penalty, the necessary energy to start the engine, must be considered, (iii) recharging the battery is possible for negative torques (braking request) or by an additional engine torque beyond the driver request torque. In the last case, the command $u_1(t)$ is larger than 1, (iv) we consider that the average efficiency of the electric motor is 0.85.

As in the general optimal control problem (1), the command variable $u(t)$ and the state variable $x(t)$ are submitted to several constraints :

- the engine can only produce a positive torque, and is limited to a maximal torque which depends on engine speed $\omega(t)$: $0 \leq T_e(t) \leq T_e^{max}(\omega(t))$,
- the electric motor torque is bounded: $T_m^{min}(\omega(t)) \leq T_m(t) \leq T_m^{max}(\omega(t))$,
- the storage capacity implies a minimal and a maximal state of charge of the battery (which are fixed to 50% and 70% in our study): $x_{(1)min} \leq x_1(t) \leq x_{(1)max}$,

- the final state of charge is constrained to be close to the initial state of charge to maintain a null electrical energy balance (to avoid to discharge totally the battery for minimizing the consumption) $x_1(0) = x_1(T)$.

Constraints on command $u(t)$ are directly derived from the constraints on maximal and minimal engine and electric motor torque

$$u(t)T_{rq}(t) \leq T_e^{max}(t), \quad (7)$$

$$T_m^{min}(t) \leq (1 - u(t))T_{rq}(t) \leq T_m^{max}(t). \quad (8)$$

In the following, we will note $U(t)$ (respectively U_k in discrete time) the feasible domain of $u(t)$ (constraints (7) and (8)).

5. Dynamic Programming

The Dynamic Programming method (DP) is classically applied to solve the problem (1) ([3], [4]) : it relies on a very simple idea, the principle of optimality, stated by Richard Bellman. It states that an optimal policy can be built step by step, first by building an optimal control for the "tail subproblem" involving the last step, then by extending the optimal policy to the "tail subproblem" involving the last two stages, and by iterating until an optimal control has been built for the complete time interval. Thus, the optimization problem is solved backward from final time to initial time.

The optimal control problem (1) is thus discretized in time:

$$\min_{u \in U} J(u) := \sum_{k=0}^{N-1} L_k(x_k, u_k) \Delta t + g(x_N), \quad (9)$$

where $L_k(x_k, u_k)$ is the cumulated fuel consumption over the time interval $[k, k + 1]$, x_k is the state vector at time k , and N is the final time of the driving cycle. The constraint on the final state of charge $x_0 = x_N$ has been introduced via a penalization term $g(x_N) = \beta(x_N - x_0)^2$ (β , a constant to be chosen).

From Bellman principle, the minimum cost-to-go $V_k(x_k)$ for the time interval $[k, N]$, $0 \leq k \leq N - 1$, is then expressed as follows :

$$V_k(x_k) = \min_{u_k \in U} (L(x_k, u_k) \Delta t + V_{k+1}(f_k(x_k, u_k))), \quad (10)$$

where f_k corresponds to the system dynamic, given by:

$$x_{k+1} = f_k(x_k, u_k) = \begin{pmatrix} x_{(1)k} + f_{(1)k}(u_k, k) \\ u_{(3)k} \end{pmatrix} \quad (11)$$

f_k being the function that modelizes the battery state of charge dynamic in the discrete form of (6), as well as the engine state. At time N , the cost function is

$$V_N(x_N) = g(x_N). \quad (12)$$

This optimization problem is solved backward from final time step to initial time step using a discretization of function V in the control space and in the state space, that gives a 3-D grid (time t , state of charge x_1 , and engine state x_2 nodes). A standard time step used in our examples is $\Delta t = 0.5s$, the discretization step Δx_1 for the state x_1 is 0.25 %, and x_2 takes only 0 or 1 values. Thus, the number of state nodes is $M = 2((x_{(1)max} - x_{(1)min})/\Delta x_1)$.

A standard DP algorithm consists in computing, at each step k and at each state value, all feasible controls, and calculating the optimal control that minimizes the sum $(L_k(x_k, u_k) + V_{k+1}(f_k(x_k, u_k)))$.

This method allows a fast optimization: $(N - 1)M$ constrained optimization problems (10) of only one parameter u_k are solved with a fine time discretization of the controller. This provides the optimal power repartition u_k^* at each time step (we refer to [9] for some theoretical results on the convergence of the method and error estimations).

A direct constrained optimization of (9) is an alternative method but leads to a large non linear optimization problem with a large number of inequality constraints. This is the reason why the DP algorithm is often preferred when the number of state variables is small (one or two state variables).

6. Variations of the size of some powertrain elements

On conventional vehicles, the maximal engine power is mainly defined by its capacity. Economic vehicles have a small engine, with a small fuel consumption, but with a low maximal torque. When engine capacity is higher, fuel consumption is higher, but maximal engine torque is also higher.

In hybrid vehicles, engine capacity has still almost the same impact on fuel consumption. However, the presence of a second energy source, which implies at least an electric motor and a battery, gives other degrees of freedom to decrease the fuel consumption.

The following results have been obtained by testing different sizes of elements of the powertrain. This approach has been preferred to parametric optimization, which is a quite complicated problem. The reader can nevertheless see some examples in [10].

6.1. Reference case

The reference case is a Citroën C1 with a 1000 cc gasoline engine. We consider on this vehicle that an electric motor of 10 kW is located downstream of the gearbox, at the same level as the wheels, and that a battery of 1.3 kWh provides the electric energy to the motor. The potential of this vehicle is assessed on the NEDC cycle, and gives the following results:

Table 1: Fuel consumption for the reference case

	Fuel consumption (l/100km)	Gain (%)
Conventional vehicle	4.76	
HEV	3.76	20.99 %

6.2. Variations of battery capacity with constant battery weight

The battery capacity affects the constraints on the state of charge: x_{min} and x_{max} . As the system dynamic does not depend on the state x , the fuel consumption will not evolve with the battery capacity as soon as the state of charge remains inside its bounds x_{min} , x_{max} without touching them. Table 2 gives optimal fuel consumption on NEDC cycle for varying capacities.

Table 2: Study of the impact of the battery capacity

	Fuel consumption (l/100km)	Gain (%)
Conventional vehicle	4.76	
HEV with battery 0.325 kWh	3.78	20.40 %
0.65 kWh	3.76	20.99 %
1.3 kWh	3.76	21 %
2.6 kWh	3.76	21 %
5.2 kWh	3.76	21 %

6.3. Variations of battery capacity and battery weight

By considering the variations of battery capacity and the corresponding variations of its weight, not only the available electric power is changed, but also the global weight of the vehicle. In this context, the necessary power to match the requested vehicle speed of the cycle becomes higher as the weight of the battery increases. Thus, the variation of battery capacity causes the engine to change its operating points: same engine speeds, but different requested torques. Also, the vehicle will potentially be able to

recover much more electric energy during braking, if control constraints are not activated. Consequently, the analysis of these results is not trivial.

We consider in the following that the battery weight varies according to 1kg for 50Wh. The Table 3 shows obtained results.

Table 3: Study of the impact of the battery capacity with weight variations

	Fuel consumption (l/100km)	Gain (%)
Conventional vehicle	4.76	
HEV with battery 0.325 kWh	3.78	20.40 %
0.65 kWh	3.76	20.84 %
1.04 kWh	3.76	21.01 %
1.3 kWh	3.76	20.99 %
1.625 kWh	3.76	20.90 %
1.95 kWh	3.76	20.79 %

It is noticeable that those results show a different trend than the results of Table 2. An optimal value is reached for $n_{capa} = 1.04$ kWh, as seen on Figure 2.

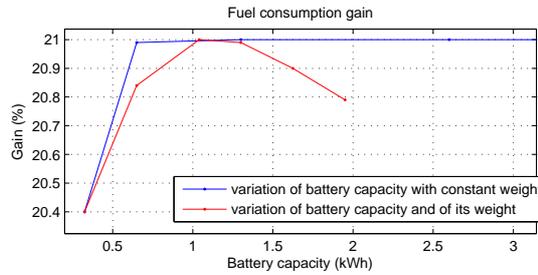


Figure 2: Variation of gain of fuel consumption with battery capacity

6.4. Variations of maximal and minimal electric motor torques

The car manufacturers are interested in estimating the best size of an electric motor for a hybrid vehicle, according to its use, as there is no need to have a very powerful motor if half of its maximum power is never reached.

The variations of maximal and minimal electric motor power may change the fuel consumption, but it also changes the state of charge trajectory (see results in Table 4).

Table 4: Study of the impact of the maximal motor power

	Fuel consumption (l/100km)	Gain (%)
Conventional vehicle	4.76	
HEV with motor P= 5 kW	3.80	20.15 %
HEV with motor P= 10 kW	3.76	20.99 %
HEV with motor P= 15 kW	3.74	21.25 %
HEV with motor P= 20 kW	3.74	21.25 %

For this vehicle used on the NEDC cycle, the fuel consumption does not evolve anymore if the electric motor power is larger than $P = 15$ kW. Smaller powers of electric motor do not affect much the fuel consumption, as the gain for an electric motor is still 20.15% for a power of 5 kW.

7. Real-time control strategy

7.1. Validation of DP results on a more realistic model

Results obtained with the dynamic programming applied on the matlab model are tested on a more realistic model implemented in AMESim, corresponding to the reference case, see Figure 3.

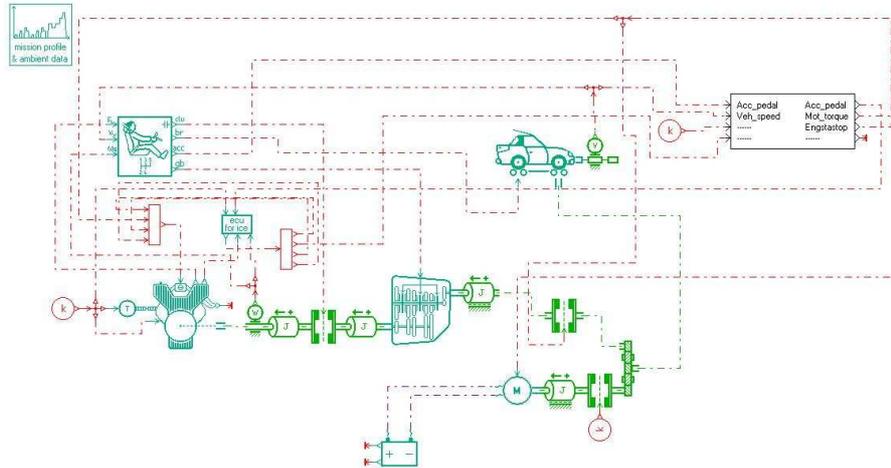


Figure 3: Validation AMESim model

By comparison with the matlab model used for optimization, this AMESim model includes all axle inertias, lag times of engine and electric motor, and a driver model which controls acceleration and brake pedal to follow the speed request from the cycle. Figure 4 shows the behavior of AMESim model with the optimal control of electric motor obtained from dynamic programming on the Matlab model.

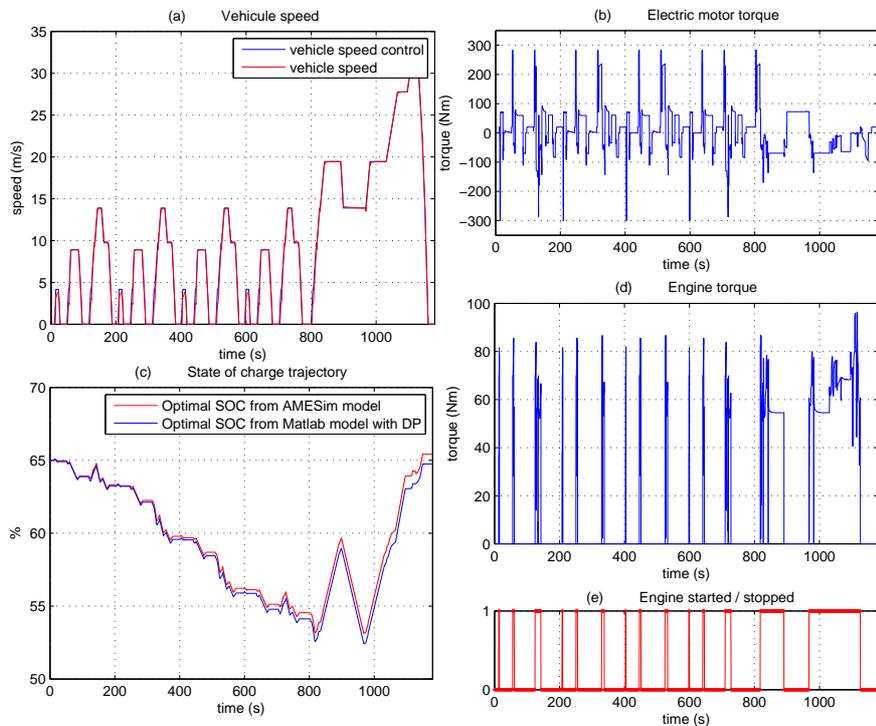


Figure 4: Results of optimal control applied on AMESim model

Figure 4(a) shows the effective vehicle speed (blue), compared with the speed request from the cycle (red). Figure 4(b) and (d) show the motor and engine torques, while Figure 4(e) specifies if the engine is started or stopped. Figure 4(c) shows a comparison between optimal SOC trajectory obtained with matlab model, and the one obtained with AMESim model.

The global fuel consumption obtained with AMESim model is 310.2 g (3.81 l/100km) while the one with matlab model is 310.34 g (3.75 l/100km), but the vehicle linear displacement is not the same: Matlab model covers 11.01km, but AMESim model only covers 10.84km, because of inertia that cause a bad follow-up of the vehicle cycle at low speeds. The resulting error between the two models is about 1.5%.

7.2. Real-time control strategy

In this section, suboptimal control law is derived from the optimization results. The chosen real-time control law is the Equivalent Consumption Minimization Strategy: it is based on instantaneous equivalent fuel consumption, the battery being considered as a second energy source. ECMS aims to choose the cheapest energy source, between fuel tank and battery storage.

This strategy is derived from the Pontryagin's Principle, which set optimality conditions for the continuous optimization problem (1). Some important theoretical results can be found in [8]. To apply this strategy, we form the Hamiltonian function \mathcal{H} given by

$$\mathcal{H}(x(t), u(t), p(t), t) = \hat{L}(x(t), u(t), t)\Delta t + p(t)\dot{x}_1(t)\Delta t, \quad (13)$$

where $\hat{L}(x(t), u(t), t)\Delta t$ represents the variation of fuel consumption over a time step Δt , given by

$$\hat{L}(x(t), u(t), t) = \begin{cases} L_1(u(t), t) & \text{if } x_2 \neq 0 \\ 0 & \text{if } x_2 = 0, \end{cases} \quad (14)$$

$\dot{x}_1\Delta t$ corresponds to the variation of state of charge over a time step, and p is a Lagrange multiplier associated with the constraint $\dot{x}_1(t) = f_1(u(t), t)$. The state constraints have not been considered in the definition of (13). In this strategy, the optimal control is the admissible one which minimizes \mathcal{H} , taking into account all the control constraints. The same models as the ones used in dynamic programming algorithm are used in ECMS, which is the advantage of this model-based strategy.

7.2. Meaning of p

The coefficient p corresponds to the *price* of electric energy. The larger is p , the more expensive is the electrical energy, and the more interesting to be recovered. The smaller is p , the cheaper is the electrical energy, and the more interesting to be used to power the vehicle conjointly with the engine.

The variation of p depends on the time evolution of the dynamic state variable x_1 , according to the Euler-Lagrange equations, namely

$$\dot{p}(t) = -\frac{\partial \mathcal{H}}{\partial x_1}(x(t), u^*(t), p(t), t). \quad (15)$$

As either \hat{L} or \dot{x}_1 do not depend on x_1 , Equation (15) gives

$$\dot{p}(t) = 0. \quad (16)$$

As a results, when no state constraint is active, p is constant for the considered system (6). When a state constraint becomes active, the sign of \dot{p} is completely known, depending on the active constraint: $x = x_{min}$ or $x = x_{max}$.

7.3. Control of p

In ECMS, the value of p is often controlled thanks to rules [5], [11], mainly depending of the state of charge. Indeed, as the future is not known, it can be better to keep the state of charge x_1 far from its bounds, and to control p as a function of x_1 with a feedback controller. However, in this paper, we

consider that p remains constant, and we just try to show the potential of this strategy with an adequate initial value of p .

An estimation of p can be obtained by calculating the equivalent Lagrange multiplier with the dynamic programming results. From [7], equivalence between Pontryagin's principle and dynamic programming can be found

$$p(t) = \frac{\partial V}{\partial x}(t, x), \quad (17)$$

where $V(t, x)$ corresponds to the cost-to-go function in continuous time, already defined in (10). Using these results allows to estimate that p should be into the range $[-3400; -3000]$. Because of some differences between the model used with dynamic programming and the AMESim model, a trial & error method has been applied to end up with $p = -3075$ [†]. This value allows to reach a final state $x(T)$ close to the initial state $x(0)$.

7.4. Results of the ECMS Real-time controller

The Figure 5 shows some results obtained with this real-time control strategy, with a correct value of p .

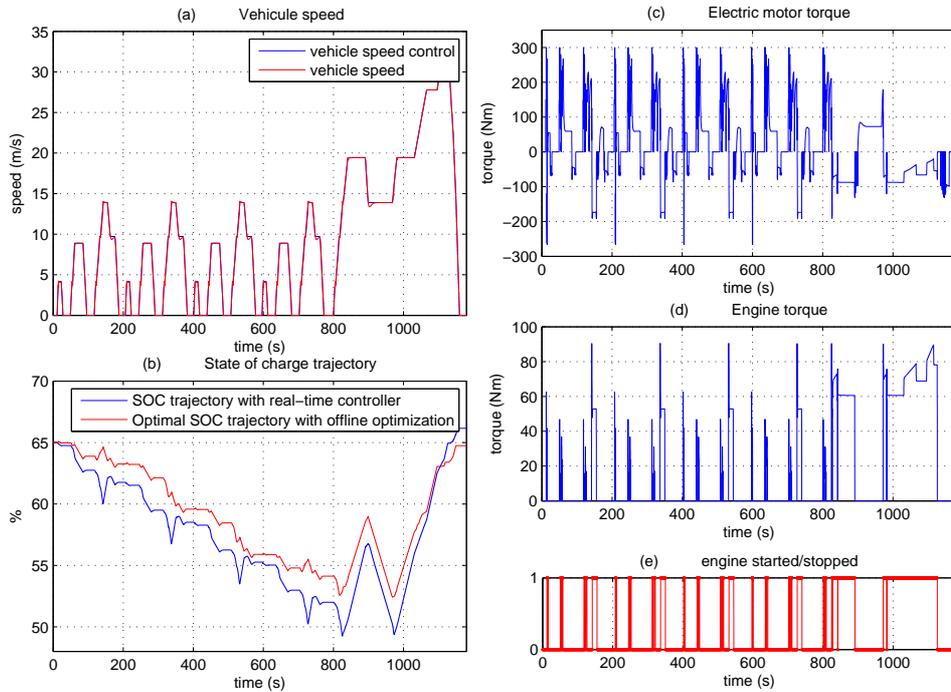


Figure 5: Results obtained with a sub-optimal strategy applied on AMESim model

Figure 5(a) shows that the hybrid vehicle still follows the speed request from the cycle. However, some differences appear on the state of charge, Figure 5(b). The fuel consumption obtained with this sub-optimal strategy is 326.8 g, corresponding to 3.97 l/100km. Compared to conventional vehicle, this controller – with an ideal value of p in this example – allows to save 16.6% of fuel on the NEDC cycle. Some rules to control p with the state of charge should be implemented to make this controller more robust.

[†]if the variations of fuel and chemical energies are used instead of the variations $\hat{L}(u(t), t)\Delta t$ and $\dot{x}_1(t)\Delta t$, the coefficient p can be scaled to a smaller value.

8. Conclusions

In this paper, has been presented an optimization study applied on a full hybrid vehicle, with classical offline optimization tool, and an example of a real-time control strategy.

A simple model in Matlab has been used with a dynamic programming algorithm. This allows to get the optimal sizes and powers of elements of the hybrid powertrain. Nevertheless, these results are obtained on the NEDC cycle, so the optimal sizes can be a little different on other cycles.

After a validation of the optimal trajectory and torques applied on a more realistic model in AMESim, a model-based sub-optimal controller has been implemented, using Pontryagin's Principle and optimality conditions. Even if an average value of p can be estimated from offline optimization results, the choice of an adequate initial value remains a delicate task, and p should be controlled regarding to the evolution of the state of charge of the battery.

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