Hybrid electric vehicles: from optimization toward real-time control strategies

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HYBRID ELECTRIC VEHICLES : FROM OPTIMIZATION TOWARD REAL-TIME CONTROL STRATEGIES

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Abstract: Hybrid-electric vehicles appear to be one of the most promising technologies for reducing fuel consumption and pollutant emissions. The presented work focuses on two types of architecture : a mild hybrid and a full hybrid where the kinetic energy in the breaking phases is stored in a battery to be re-used later via the electric motor. This additional traction power allows to downsize the engine and still fulfill the power requirements. Moreover, the engine can be turned off in idle phases for both architectures and for the parallel architecture, it may be turned off whereas the electric motor furnishes all the traction power. The optimal control problem of the energy management between the two power sources is solved for given driving cycles by a classical dynamic programming method and by an alternative method based on Pontryagin Minimum Principle. The real time control laws to be implemented on the vehicle are derived from the resulting optimal control strategies. These control laws are evaluated on another driving cycle which was not given a priori.

Keywords: Hybrid vehicle, Optimal control, Dynamic programming, Pontryagin, Control strategies

1. INTRODUCTION

Growing environmental concerns coupled with concerns about global crude oil supplies stimulate research on new vehicle technologies. Hybrid-electric vehicles appear to be one of the most promising technologies for reducing fuel consumption and pollutant emissions (German, 2003) : mainly thanks to the system *stop'n go* that allows to turn off the engine in idle phases, to the recuperated braking energy to be stored in a battery and re-used later via the electric motor and to the possibility to downsize the engine.

The energy management of hybrid power trains requires then some specific control laws : they rely on the estimation of the battery state of charge which provides the remaining level of energy, and the variable efficiency of each element of the power train has to be taken into account. Optimization of energy management strategies on given driving cycles is often used to derive sub-optimal control laws to be implemented on the vehicle (see among others (Sciarretta *et al.*, 2004), (Scordia, 2004), (Wu *et al.*, 2002), (Delprat, 2002)).

IFP, in partnership with Gaz de France and the Ademe, has combined its downsizing technology with a natural gas engine in a small urban demonstrator vehicle (VEHGAN vehicle), equipped with a starter alternator and supercapacitor manufactured by Valeo (Tilagone and Venturi, 2004).

In this paper, we present two different optimization algorithms and apply them to a simplified model of the VEHGAN vehicle and to a parallel architecture version of this vehicle: a classical Dynamic Programming algorithm ((Wu *et al.*, 2002), (Scordia, 2004), (Sciarretta *et al.*, 2004)), and an original algorithm based on Pontryagin Minimum Principle that allows to handle constraints on the state and control variables. Finally, we propose two types of control strategies derived from the optimization results on given driving cycles and evaluate them as a real time strategy on a driving cycle which was not given a priori.

2. SYSTEM MODELLING AND OPTIMAL CONTROL PROBLEM

2.1 Characteristics of the considered hybrid vehicle

Two different architectures are modelled:

- a mild hybrid architecture : the engine can not be stopped when the requested torque is provided only by the electric motor, except for the *stop'n go* mode at the idle speed. So, for a control that cancels the engine torque and for positive torque request, the fuel consumption does not vanish (Figure 1),
- a full parallel hybrid architecture : the engine can be stopped to let the electric motor power alone the vehicle. In that case, the fuel consumption vanishes.

In both cases, the battery is regenerated in braking phases accordingly to the available minimum electric torque at the considered engine speed.

In order to solve the optimal control problem of energy management, we build a simplified model which is composed of :

- a driving cycle to be followed (imposing vehicle speed and gear shifts),
- a vehicle model defining its mass, wheel inertia, resistance force,
- a manual gearbox with 5 gear ratios,
- a 660CC natural gas engine characterized by a fuel consumption map displayed in Figure 1 and a maximum torque depending on the engine speed (see (5)),
- a starter alternator (3kW for mild-hybrid, 6kW for full-hybrid) characterized by a maximum torque and a minimum torque for regenerative braking phases, both depending on the engine speed (see (6)). Its efficiency is assumed to be 1 in the presented examples,
- a battery characterized by a capacity of 0.4Ah for mild-hybrid architecture and 40Ah for full-hybrid one. The variations of the battery state of charge are modelled by

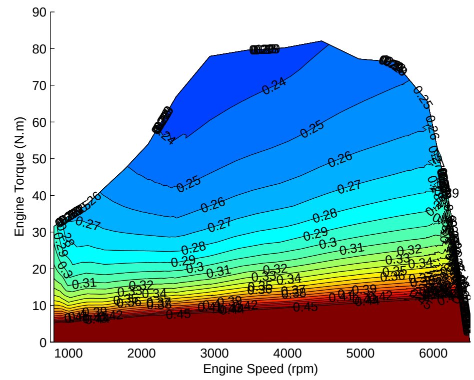

Fig. 1. Fuel consumption map of natural gas engine of VEHGAN vehicle

$$\dot{x}(t) = -\frac{\omega(t)T_m(t)K'}{U_{batt}n_{capa}} \qquad (1)$$

with $\omega(t)$, the electric motor and engine speed (assumed to be equal), U_{batt}, the battery voltage considered to be constant, K', a scaling constant and n_{capa}, the nominal capacity of the battery.

The driving cycle is converted in a (engine speed, torque) trajectory either thanks to a backward model based on the vehicle model, or thanks to a forward model as in AMESim Drive library which furnishes a more realistic trajectory taking into account a simulated behavior of a driver as the anticipation of the driving cycle.

2.2 Optimal Control Problem

The optimal control problem under study consists in minimizing the fuel consumption of the vehicle along a given driving vehicle cycle, taking into account physical constraints from battery, engine and electric motor. The control variable associated with this problem is called $u(t)$. It represents the distribution of the requested torque T_{rq}, between the engine torque T_e and the electric motor torque T_m, written as

$$\begin{cases} T_{rq}(t) = T_e(t) + T_m(t) \\ T_e(t) = u(t)T_{rq}(t) \\ T_m(t) = (1 - u(t))T_{rq}(t). \end{cases} \qquad (2)$$

The state variable is the battery state of charge $x(t)$ and follows from (1)

$$\dot{x}(t) = -K\omega(t)(1 - u(t))T_{rq}(t) = f(u(t), t), \qquad (3)$$

where $K = \frac{K'}{U_{batt}n_{capa}}$.

The resulting optimization problem is then the following :

$$\begin{cases} \min_u \left\{ J(u) = \int_0^T L(u(t),t)dt + g(x(T),T) \right\} \\ \text{subject to :} \quad \dot{x} = f(u(t),t), \quad x(0) = x_0 \\ x_{min} \le x(t) \le x_{max} \\ u_{min}(t) \le u(t) \le u_{max}(t) \end{cases} \quad (4)$$

with 0 and T, respectively the initial and the final times of the given driving cycle, $L(u(t),t)$, the instantaneous fuel consumption, computed from the map displayed in Figure 1, $g(x(T),T)$, the penalization term that constrains the final state of charge to be close to the initial state of charge in order to maintain a null electrical energy balance (to avoid to discharge totally the battery for minimizing the consumption).

The bound constraints on the state and on the control in (4) are derived from the following constraints :

- the engine can only produce a positive torque, and is limited to a maximum torque which depends on engine speed $\omega(t)$, written as $0 \le T_e(t) \le T_e^{max}(\omega(t))$, and leads to

$$0 \le u(t)T_{rq}(t) \le T_e^{max}(\omega(t)), \quad (5)$$

- the electric motor torque is limited between a maximum torque and a minimum torque during regenerating breaking, $T_m^{min}(\omega(t)) \le T_m(t) \le T_m^{max}(\omega(t))$, and leads to the control constraints

$$T_m^{min}(\omega(t)) \le (1-u(t))T_{rq}(t) \le T_m^{max}(\omega(t)), (6)$$

- the storage capacity implies a minimum and a maximum state of charge of the battery (which are fixed to 0% and 100% in our example)

$$x_{min} \le x(t) \le x_{max}. \quad (7)$$

In this optimal control problem, we make several assumptions

- the pollutant emissions are not taken into account in the optimization process,
- the engine speed and the electric motor speed are equal,
- in the mild hybrid case, recharging the battery is only possible for negative torques (breaking request), we did not consider regeneration by an additional engine torque beyond the driver request torque. Thus the control $u(t)$ remains between 0 and 1. In the full hybrid case, $u(t)$ can take values larger than 1, allowing battery regeneration with additional engine torque.

In the following, we will call $U(t)$ in continuous time (respectively U_k in discrete time) the feasible domain for $u(t)$ (respectively u_k) with respect to the constraints (5) and (6).

3. DYNAMIC PROGRAMMING OPTIMIZATION

The Dynamic Programming method (DP) is classically used to solve the problem (4) ((Wu *et al.*, 2002), (Scordia, 2004)) : it relies on the principle of optimality or Bellman principle. First, the optimal control problem (4) is discretized in time

$$\begin{cases} \min_{u_k \in U_k} J(u) := \sum_{k=0}^{N-1} L_k(u_k) + g(x_N) \\ \text{subject to :} \quad x_{k+1} = f_k(x_k,u_k), \quad x(0) = x_0 \\ x_{min} \le x_k \le x_{max} \end{cases} \quad (8)$$

where $L_k(u_k)$ is the cumulated fuel consumption over the time interval $[k, k+1]$, x_k is the state of charge of the battery at time k, f_k is the function that modelizes the battery state of charge evolution in the discrete form of (3) and $g(x_N) = \beta.(x_N - x_0)^2$ is the penalization term for the constraint on final state of charge (β is a constant to be chosen[1]), N being the final time of the driving cycle.

From Bellman principle, the minimum cost $V_k(x_k)$ at the time step k, $0 \le k \le N-1$, is expressed as

$$V_k(x_k) = \min_{u_k \in U_k} (L_k(u_k) + V_{k+1}(f_k(u_k))). \quad (9)$$

At time N, the cost function is $V_N(x_N) = g(x_N)$.

This optimization problem is solved backward from final time step to initial time step using a discretization of function V in the control space and in the state space.

3.1 DP Optimization algorithm

A standard time step used in our examples is $1s$, and the step for state discretization is 0.5%. Two algorithms may be used to solve the DP problem :

- a classical DP algorithm, called Ford algorithm in the following (Scordia, 2004), consists in exploring all the feasible controls (to go from a point x_k^i to an other point x_{k+1}^j), finally taking the best trajectory (the trajectory which minimizes at each step k the sum $L_k(u_k) + V_{k+1}(f_k(u_k))$). In such a method, the state of charge trajectory remains on the points of the defined grid in the state space which may lead to inaccurate results.
- the chosen algorithm interpolates the function $V(x_k, k)$ in the state space, for each time step k thanks to an upwind scheme (Guilbaud, 2002) :

[1] In the following results, a value depending of battery capacity has been implemented

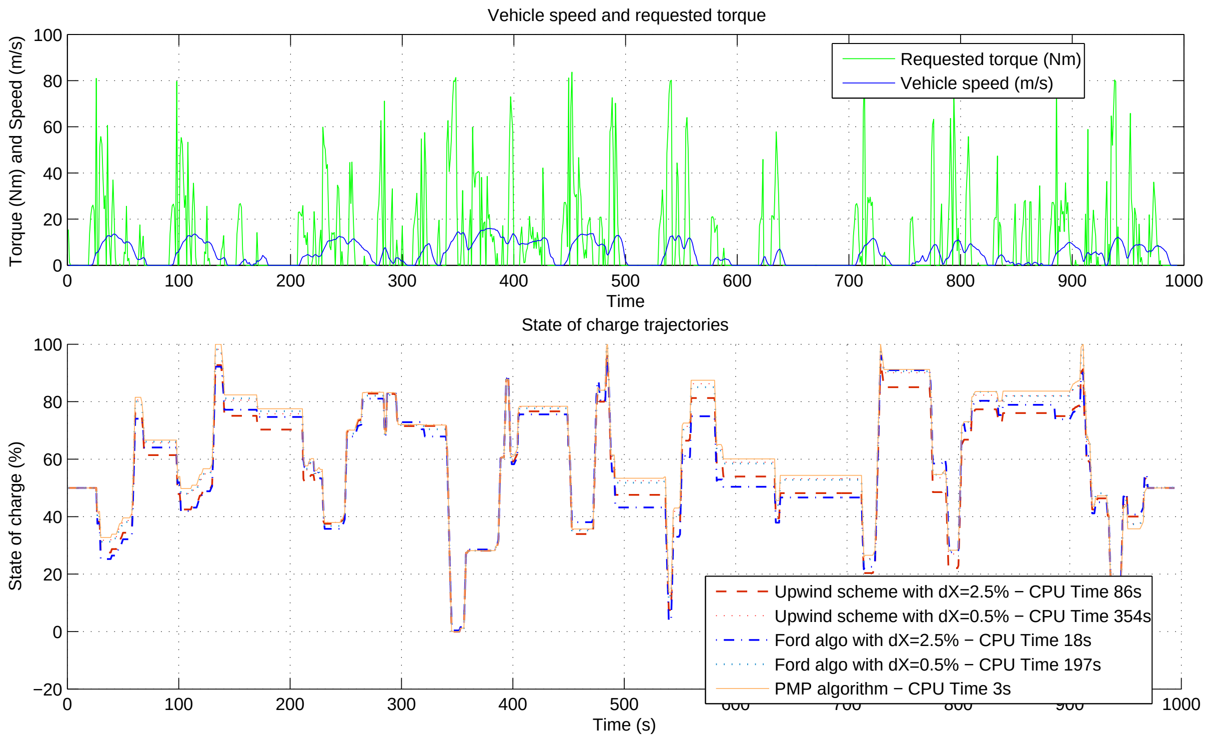

Fig. 2. Urban Artemis cycle (*Top*); Optimal state of charge trajectory of VEHGAN vehicle computed with PMP & DP algorithm (*Bottom*).

$$V_k(x_k^i) = \min_{u_k \in U_k} [\Delta t L_k(u_k) + V_{k+1}(x_{k+1}^i)$$
$$+ f_k(u_k) \frac{V_{k+1}(x_{k+1}^i) - V_{k+1}(x_{k+1}^{i-1})}{\Delta x} \Delta t], \quad (10)$$

where Δx and Δt are respectively the state and the time discretization step size. We refer to (Guilbaud, 2002) for some theoretical results on the convergence of this method and error estimations. Therefore, it is possible to use a (state) continuous constrained optimization algorithm to solve each problem (9) which should furnish more accurate results than Ford algorithm. Nevertheless, this algorithm is generally more expensive in terms of computing time.

These two optimization algorithms are only used when $T_{rq} > 0$: when the requested torque is negative, the optimal control u_k is completely known, as the battery is regenerated as much as possible, the control u_k being constrained by the minimal electric motor torque from (6) and by maximum SOC from (7).

Optimization results obtained with DP method are displayed on Figure 2.

4. PONTRYAGIN MINIMUM PRINCIPLE OPTIMIZATION

In this section, we propose an alternative method to solve the optimal control problem (4). It relies on the Pontryagin Minimum Principle (PMP) and unlike the DP method does not require any discretization scheme.

4.1 Pontryagin Minimum Principle

First we consider the optimization problem (4) and introduce the Hamiltonian function, without considering state and control constraints

$$\mathcal{H}(u(t), x(t), p(t)) = L(u(t), t) + p(t)\dot{x}(t). \quad (11)$$

$p(t)$ is called the co-state of our system. We assume here that L is a smooth convex function of u.

The Pontryagin Minimum Principle states the following conditions for the unconstrained optimal control problem :

$$\frac{\partial \mathcal{H}}{\partial x} = -\dot{p} \quad \text{and} \quad \frac{\partial \mathcal{H}}{\partial u} = 0. \quad (12)$$

We refer to (Pontryagin *et al.*, 1974) and (Bryson and Ho, 1975) for further details about Pontryagin Principle.

4.2 Application

The fuel consumption $L(u(t), t)$ to be minimized in (4), is defined by a discrete map $L(\omega, T_e)$, modelled by a 2-order polynomial, which is represented as

$$L(\omega, T_e) = \sum_{i,j=0}^{2} K_{ij}\omega^i T_e^j, \qquad (13)$$

which allows to model a large variety of engine maps (Rousseau *et al.*, 2006).

4.2.1. Mild-Hybrid case In the mild-hybrid vehicle case, the fuel consumption can not be cancelled. We do not consider the stop and start, as well as the possibility to power the vehicle only with the electric motor.

From (12) and (3) we obtain

$$\dot{p} = 0 \Rightarrow p = constant = p_0. \qquad (14)$$

Without any constraint on the state and on the control, the problem of minimizing H can be easily solved. The minimum fuel consumption is then reached for u^* so as

$$\frac{\partial \mathcal{H}}{\partial u} = \frac{\partial L}{\partial u} + p\frac{\partial f}{\partial u} = 0. \qquad (15)$$

The optimal control u^* can be calculated easily by solving the equation (15), which depends linearly on u (thanks to (3) and (13)) . u^* finally depends on $p(t)$, $T_{rq}(t)$ and $\omega(t)$

$$u^*(t) = -\frac{\sum_{i=0}^{2} K_{i1}\omega(t)^i + p_0.K.\omega(t)}{2\sum_{i=0}^{2} K_{i2}\omega(t)^i.T_{rq}(t)}. \qquad (16)$$

The expression of p_0 is obtained by replacing $u^*(t)$ by its expression in the state equation (3), and by integrating this equation in time, between T_{init} and τ, T_{init} and τ being respectively the considered initial and final times.

4.2.2. Full-Hybrid case With the full-hybrid case, we have to consider the possibility to power the vehicle only with the electric motor. The previous expression of Hamiltonian becomes unadapted, as the fuel consumption can be completely cancelled. The fuel consumption function is then discontinuous

$$L_{fh}(\omega(t), Te(t)) = \begin{cases} 0 & \text{if } u(t) = 0 \\ L(\omega(t), Te(t)) & \text{if } u(t) \neq 0. \end{cases} \qquad (17)$$

The Hamiltonian, in the only electric motor case ($u(t) = 0$), is then written

$$\mathcal{H}_m(x(t), p(t)) = p(t)\dot{x}(t). \qquad (18)$$

The optimal control u^* must then be written as

$$u^* = argmin[\mathcal{H}(u(t), x(t), p(t)), \mathcal{H}_m(x(t), p(t))]. \qquad (19)$$

4.2.3. Handling constraints on control and state variables The previous section presents the computation of the optimal control of the continuous problem in a restricted case where no constraint is introduced. While control constraints are generally easily taken into account, handling the state constraints in the continuous optimal control problem is cumbersome: several singular cases can be found in (Bryson and Ho, 1975). In our application, we are not able to find an analytic solution of the optimal control problem with control constraints : indeed, these constraints depends on time and depends on p_0 which depends on final SOC (cf. previous section). By an iterative method (called *algo1* in the following), we can compute the value of p_0 in order to reach the desired SOC at final time with the control, expression (16), projected on its bound constraints.

(Hartl *et al.*, 1995), (Pontryagin *et al.*, 1974), (Evans, 2000), (Bryson and Ho, 1975), (Guilbaud, 2002) have studied the general problem (4) with the state constraints. In our application, we can show that $p(t)$ presents discontinuities at the time steps where the state inequality constraints are saturated. These time steps are not a priori known : this prevents us to solve explicitly the continuous optimal control problem with these state constraints.

4.2.4. PMP Optimization algorithm Considering the difficulties described in previous section, we propose a heuristic iterative method that allows to find a sub-optimal trajectory from the constrained continuous optimal control problem (4). The proposed algorithm consists in an initialization step and 3 steps :

(0) *algo1* is applied on the driving cycle $[0, T]$ (see Figure 3 Step 0). The obtained optimal trajectory violates the state constraints, the farthest SOC (ie the "most violated point") from the bounds being for instance at point $(x(t_v) = -37\%, t_v = 818s)$. The initial time is called t_i, here set to 0.

(1) The SOC at t_v is projected on the nearest bound of the feasible state domain (for instance, SOC is fixed to $x_{min} = 0$ at point t_v).

(2) *algo1* is applied again on $[t_i, t_v]$ (see Figure 3 Step 2). If the obtained trajectory still violates the state constraints on $[t_i, t_v]$, steps 1 and 2 are applied again on the farthest SOC from the bounds (defining a new point t_v). This procedure is repeated until the trajectory remains on the feasible domain. Then

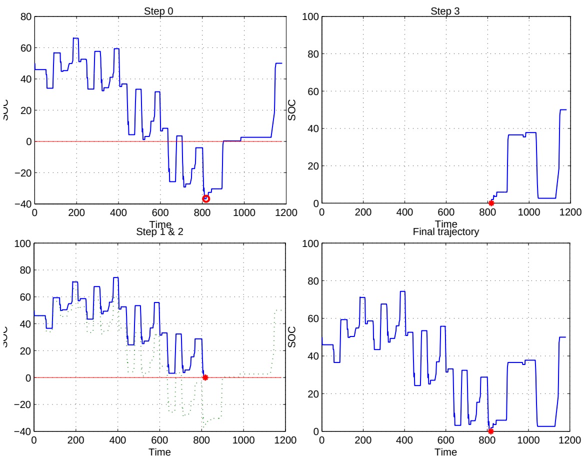

Fig. 3. The proposed algorithm based on Pontryagin Minimum Principle.

the last point t_v becomes the new initial time t_i in step 3.

(3) *algo1* is applied on $[t_i, T]$ (see Figure 3 Step 3). If the obtained optimal trajectory still violates the state constraints, steps 1 and 2 are repeated. This sequence is repeated until we reach the final step T at the desired final SOC, without violating the state constraints (Figure 3 bottom right).

4.3 Some optimization results

4.3.1. Mild Hybrid case We can compare the two optimization algorithms (DP and PMP) on the Urban Artemis driving cycle (André, 2004), in the mild Hybrid case, on Figure 2. The curves are very similar; we can notice that smaller is the state step size, nearer to the PMP curve are the DP curves.

Figure 4 presents the operating points (OP) of the engine obtained with PMP algorithm.

In this vehicle configuration, the state constraints are active 5 times, giving 6 different values of the Lagrange multiplier $p(t)$. We display the six curves (green lines) $\frac{\partial \mathcal{H}}{\partial T_e}(p) = 0$, which give optimal engine torque, function of engine speed. The engine OP are thus moved toward the green optimal curves when it is possible: the OP located below the curves remain unchanged (no battery regeneration being possible for positive torque requests for mild hybrid) whereas the OP located above are moved toward the curves by decreasing the engine torque as much as possible (saturating electric motor torque constraints).

4.3.2. Full Hybrid case Figure 5 gives optimized operating points for the engine and the electric motor (PMP algorithm is used). In addition to kinetic energy, we assume that it is possible to recharge the battery by using the engine at better OP, with an ideal efficiency of 1.

As for mild-hybrid case, the optimal trajectory (continuous green line) gives the optimal operating points of the engine by finding the solution of $\frac{\partial \mathcal{H}}{\partial T_e} = 0$. Thus, many of low torque OP are moved to the optimal trajectory, recharging the battery by imposing a negative electric motor torque. As the full-hybrid configuration allows to turn off the engine for non-zero vehicle speed (pure electric mode), most of OP associated with engine speed below 3000 rpm and requested torque below 20Nm, lead to turn off the engine (points where engine torque is zero) : turning off the engine is more efficient than the optimal engine torque (green curve : $\frac{\partial \mathcal{H}}{\partial T_e} = 0$).

5. REAL-TIME CONTROL

From optimization results on Urban Artemis cycle, we derive suboptimal control laws that will be tested on an other cycle. In this section, the FTP72 cycle has been chosen, for its realism of urban driving.

Two different control laws will be tested : the first one, based on Optimization results from Pontryagin principle, consists of varying the value of p regarding to the state of charge, to control $u(t)$, then the electric motor. The reference Lagrange multiplier value p is the mean of optimal values of p, obtained on Artemis Urban cycle with off-line optimization using PMP algorithm.
The second one uses a map of electric motor torque created by the optimization results on Urban Artemis cycle. The electric motor torque from the map is then weighted by the state of charge of the battery : reduced if the SOC is low, increased if the SOC is high. The obtained results are displayed in Table 1. For the mild hybrid configuration, the suboptimal laws give fuel consumptions which are close to the optimal one.

Table 1. Fuel Consumption

Consump. (l/100km)	Th. veh.	Optimal control	p-control based	Elec. mot. torq. map
Mild-H.	3.32	3.22 (-3,01%)	3.23 (-2,71%)	3.23 (-2,71%)
Mild-H with Stop'n go.		2.86 (-13,62%)	2.87 (-13,49%)	2.88 (-13,33%)
Full-H.		2.70 (-18.67%)	2.83 (-14,76%)	2.86 (-13,85%)

For the full hybrid architecture, the two control laws give degraded results compared to optimal results. Many reasons can explain these differences. First, even if Urban Artemis cycle and FTP72 cycle are both realistic of an urban driving, operating points are very different. While

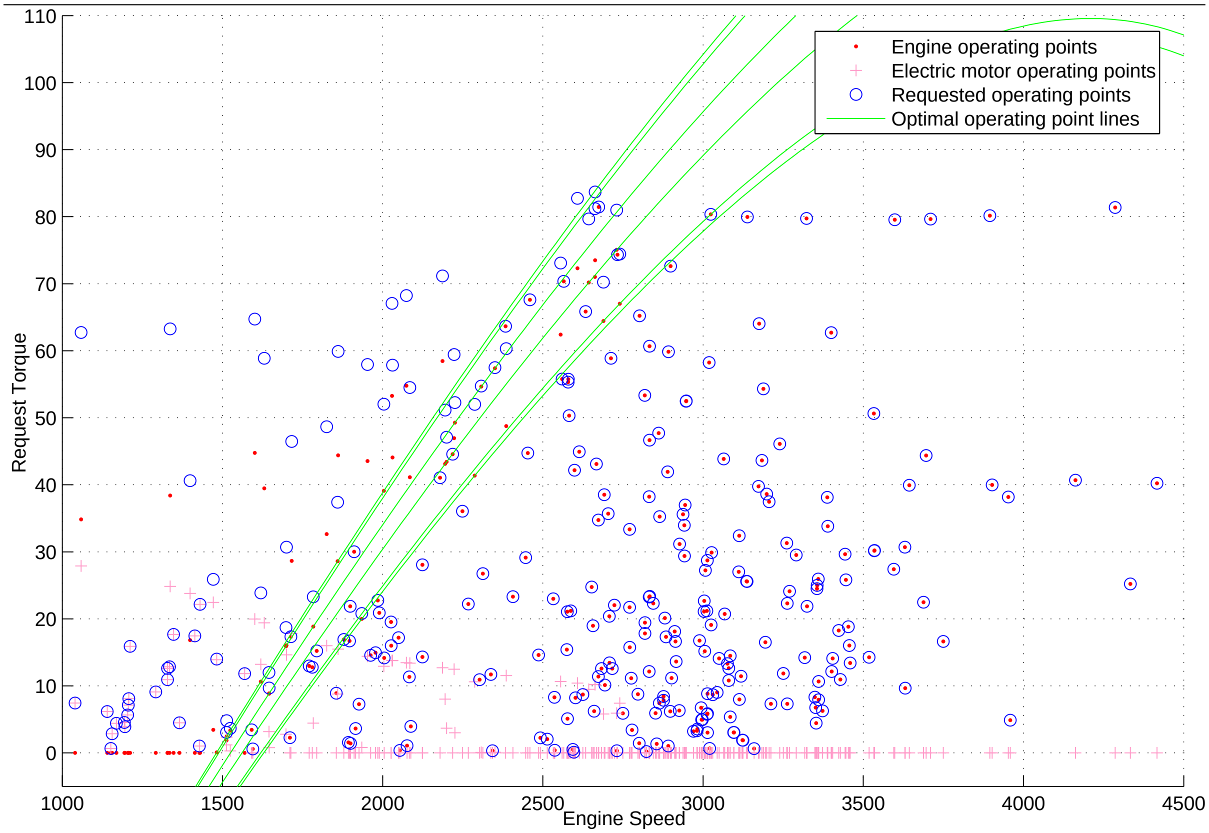

Fig. 4. Operating points of engine in Mild-Hybrid mode obtained by PMP algorithm for the urban Artemis Driving Cycle.

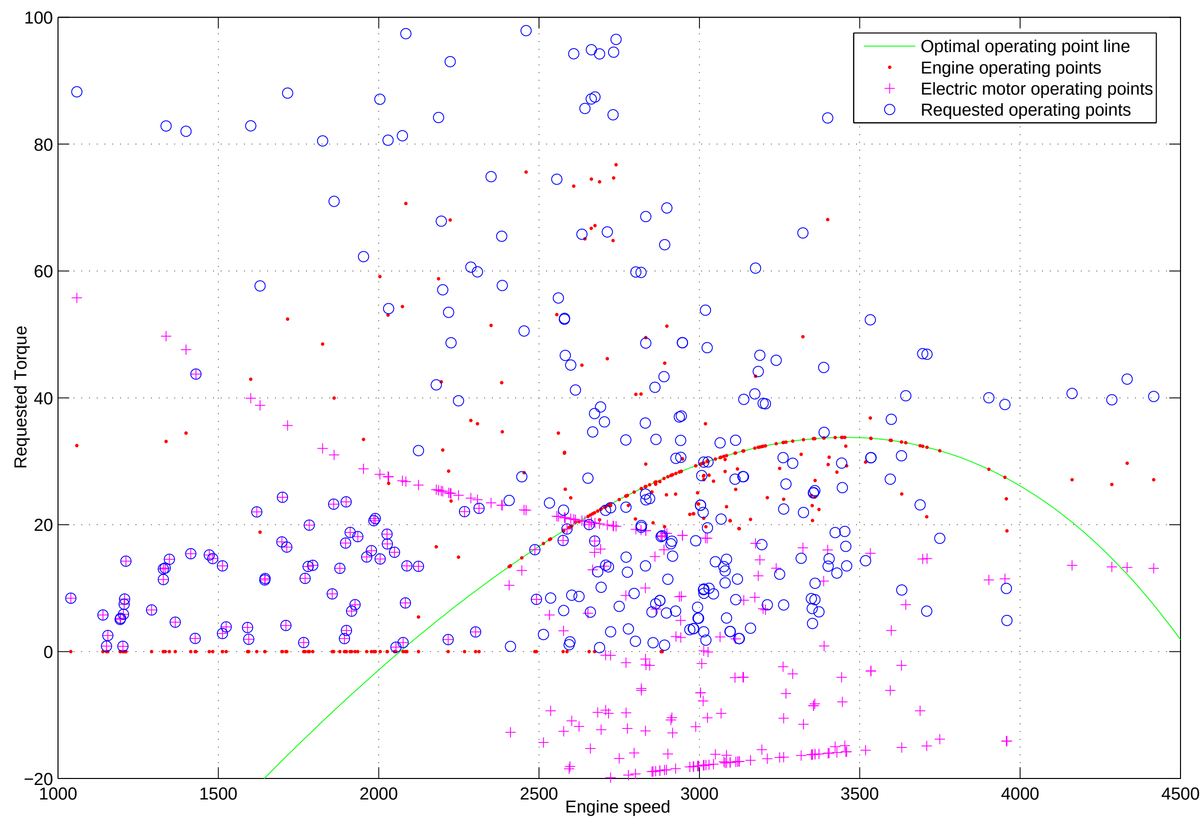

Fig. 5. Operating points of engine in Full-hybrid mode obtained by PMP algorithm for the urban Artemis Driving Cycle.

requested operating points of Artemis cycle are almost uniformly located in the whole engine speed and torque space, all requested operating points of FTP72 are below $\omega = 3200$ rpm, with a majority below $\omega = 2000$ rpm. The consequence is a unadapted electric motor map for the second control law. Concerning the first control law, the optimal p (obtained with PMP algorithm on FTP72) is quite different from the optimal p obtained for Artemis cycle, leading to degraded results.
Nevertheless, the consumption gain remains high : -14.76%.

These results illustrate that several driving cycles are needed to develop efficient suboptimal control laws based on p-control or electric motor map. The vehicle speed (related to engine speed by gear ratios) could also be taken into account to improve fuel consumption gains.

6. CONCLUSIONS

In this study, we have presented two methods for optimal control optimization. The heuristic method based on Pontryagin Minimum Principle, well known in the free state constraint case, has been applied successfully to our state constrained problem, with very similar results to Dynamic Programming methods and a computation time divided by 100. Nevertheless, there is currently no theoretical proof to confirm the presented validation results. Moreover, there are some limitations to this approach, mainly the assumptions on the fuel consumption map, modelled by a smooth convex function of control u (2-order polynomial) ; this limitation could lead to a bad approximation of the real fuel consumption for some particular engines.
Other degrees of freedom, as the gear-shifting sequence should also be taken into account in the optimization problem to improve the fuel consumption gain. Reduction of pollutant emissions will also be studied by considering a second state based on exhaust temperature.
From optimization results are derived two types of suboptimal feedback laws based on state of charge measurements. These laws give encouraging results even if it needs to be improved in the full hybrid case.

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