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A generalization of the local gradual deformation method using domain parameterization

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Abstract

Reservoir model needs to be constrained by various data, including dynamic production data. Reservoir heterogeneities are usually described using geostatistical approaches. Constraining geological/geostatistical model realizations by dynamic data is generally performed through history matching, which is a complex inversion process and requires a parameterization of the geostatistical realizations for model updating. However, the parameterization techniques are still not very efficient and need to be improved.

In recent years, the local gradual deformation method has been widely used to parameterize geostatistical realizations. The domain deformation technique has also been developed to improve the history matching efficiency. Both methods can smoothly modify model realizations while conserving spatial geostatistical properties. The first one consists in locally combining two or more realizations while the second one allows the optimization process to change the model realization via the variation of the shape of geometrical domains. In this paper, we generalize the local gradual deformation method by adding the possibility to change the geometry of local zones through the domain deformation. This generalization provides a greater flexibility in the definition of the local domains for the local gradual deformation method. In addition, we propose a new way to initialize the realization which guaranties a good initial point for the optimization and potentially improves the efficiency of history matching.

Keywords: History matching, Geostatistical realizations, Parameterization, Gradual deformation, Domain deformation

1. Introduction

A reservoir model is built based on both static and dynamic data. Static data represent the data obtained from experiments carried out on cores extracted from wells or measurements of well logs such as porosity or permeability. Dynamic data are generally the well production data such as well pressure, oil rate, etc.. The integration of dynamic data in the reservoir model is generally performed through history matching.

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Reservoir heterogeneities are described using geological/geostatistical approach. The uncertainty of a model realization is linked to the geological scheme, to the sedimentary concept, to the nature of the reservoir rocks, their extent, and to their properties. Ignoring the uncertainties in the reservoir lithology would lead to underestimating the complexity of the reservoir between wells, resulting in over or under-estimating the connected reservoir pore volumes. Integration of dynamic data to constrain the geostatistical realization can reduce greatly model uncertainties. Although reservoir heterogeneities are commonly generated using geostatistical models, random realizations cannot generally match observed dynamic data. To constrain model realizations to reproduce measured dynamic data, an optimization procedure may be applied in an attempt to minimize an objective function. Such history matching methods require a parameterization of the geostatistical model to allow the updating of an initial model realization.

To parameterize the geostatistical model, several methods were introduced. For example, the pilot point method (Marsily et al., 1984), the gradual deformation method (Roggero & Hu, 1998), the domain deformation method (Ding & Roggero, 2010) and the probability perturbation method (Hoffman & Caers, 2003) were all proposed to continuously deform the models. All these methods allow the modification of a geostatistical realization by preserving its spatial variability.

In recent years, the local gradual deformation method has been increasingly used (Roggero et al., 2007, Al-akhdar et al., 2012). In that method, the deformation zones are fixed and cannot be changed during history matching. If these zones are not suitably defined, it is difficult to decrease the objective function for history matching and to find an optimal realization. The choice of deformation zones is a critical point for the successful history matching of geostatistical realizations. In this paper, we propose a generalization of the local gradual deformation method, which can optimize the deformation zones through the domain deformation technique during a history matching process. This method allows the gradual deformation in varying domains and find more efficiently an optimal geostatistical realization.

Another issue in history matching is the selection of the initial model realization for the local gradual deformation. The batchwork method, which combines locally different realizations according to the matching results on the wells (Reis et al., 2000), is sometimes used to define an initial model. But this method does not always work well, and sometimes gives very bad results. Using the domain deformation technique alone may provide a suitable approach to define a convenient initial model.

In this paper, we will first briefly review the gradual deformation and the domain deformation methods, then present a generalization of the gradual deformation method by combining with the domain deformation technique to modify the shapes and sizes of the domains. We will also show how to get a convenient initial model for history matching by using the domain deformation technique. Examples are presented to show some promising results with the new technique.

2. A generalized local gradual deformation technique

2.1. Gradual deformation method

Some geostatistical methods such as the Fast Fourier Transform Moving Average (FFT-MA) (Le Ravalec et al., 2000) allow us to uncouple uncorrelated random realizations from structured information (mean, variance, correlation length, etc). With such methods, a model realization M is linked to a standard Gaussian white noise Z by an operator G:

$$M = G(Z) \tag{1}$$

The gradual deformation method (Gervais et al., 2007, Hu, 2002, Roggero & Hu, 1998) consists in combining two or more Gaussian white noises to modify the model realization. More precisely, it uses the fact that if $(Z_0, \ldots, Z_N)$ are $N+1$ independent standard Gaussian white noises and $(a_0, \ldots, a_N)$ are $N+1$ real numbers such that $\sum_i a_i^2 = 1$, then $Z = \sum_i a_i Z_i$ is still a standard Gaussian white noise. Then, if the a_i depend on a set of parameters $\rho = (\rho^1, \ldots, \rho^N)$ that guaranties that for all ρ, $\sum_i a_i^2(\rho) = 1$, we can generate new model realizations for all ρ. Moreover, a continuous variation of the set of parameters ρ gives a continuous variation of the spatial properties of the model realization as illustrated in Fig.1. For example,

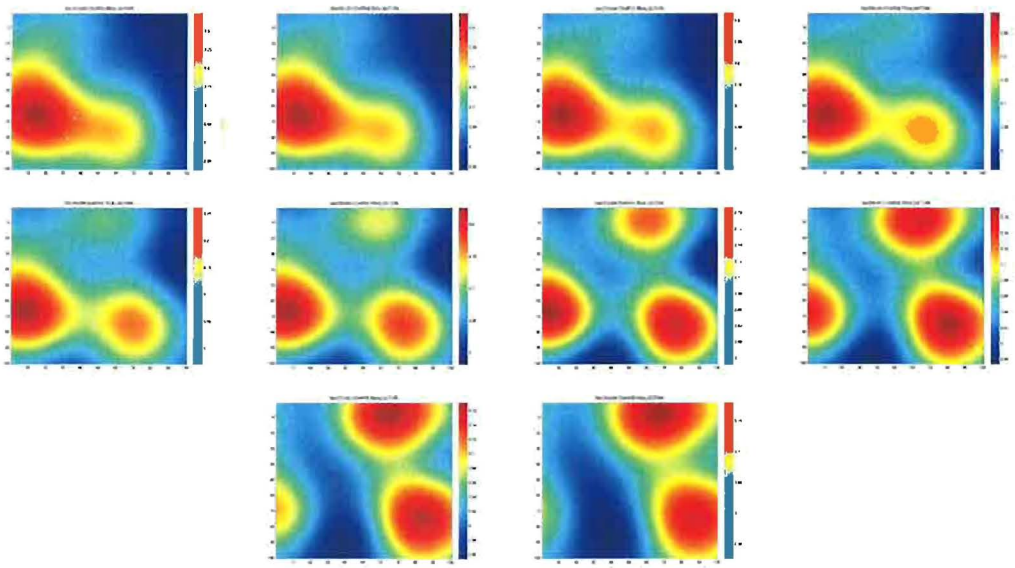


Figure 1: Model variation with the variation of a local gradual deformation parameter ρ^1

if we want to combine two independent Gaussian white noises Z_0 and Z_1, we can introduce a parameter ρ^1 and choose $Z(\rho^1)$ such as:

$$Z(\rho^1) = \cos(\rho^1) Z_0 + \sin(\rho^1) Z_1 \tag{2}$$

To combine $N+1$ Gaussian white noises $(Z_0, \ldots, Z_N)$, we introduce N parameters $\rho = (\rho^1, \ldots, \rho^N)$. The gradual deformation is then given by:

$$Z = \prod_{i=1}^{N} \cos(\rho^i) Z_0 + \sum_{i=1}^{N-1} \sin(\rho^i) \prod_{k=i+1}^{N} \cos(\rho^k) Z_i + \sin(\rho^N) Z_N \tag{3}$$

For local gradual deformation, we group the model grid cells in zones and locally combine several Gaussian white noises inside these zones. For example: if the model is divided into 2 zones and 3 standard Gaussian white noises are available:

$$Z_0 = \begin{bmatrix} Z_{0,zone_1} \\ Z_{0,zone_2} \end{bmatrix}, Z_1 = \begin{bmatrix} Z_{1,zone_1} \\ Z_{1,zone_2} \end{bmatrix} \text{ and } Z_2 = \begin{bmatrix} Z_{2,zone_1} \\ Z_{2,zone_2} \end{bmatrix} \tag{4}$$

we can define $Z(\rho)$ by :

$$Z(\rho) = \begin{bmatrix} \cos(\rho^1) Z_{0,zone_1} + \sin(\rho^1) Z_{1,zone_1} \\ \cos(\rho^2) Z_{0,zone_2} + \sin(\rho^2) Z_{2,zone_2} \end{bmatrix} \tag{5}$$

with $\rho = (\rho^1, \rho^2)$. $Z(\rho)$ is still a standard Gaussian white noise and can thus still legitimately be used to generate a model realization through the operator G. In this case, Z_0 and Z_1 are combined in the domain $zone_1$ and Z_0 and Z_2 are combined in the domain $zone_2$. This approach allows us to independently modify realizations in several regions of the model.

2.2. Domains deformation method

The domain deformation method (Ding & Roggero, 2010) has some similarities to the local gradual deformation method. The model is divided into different zones (not necessarily delimited by grid cells) and a standard Gaussian white noise is restrained to each zone. The geostatistical model realization is then modified by deforming the shapes and sizes of the zones. Fig.2 shows an example of a reservoir model divided into two domains R_1 and R_2. We build the model realization with a standard Gaussian white noise associated to Z_1 inside R_1 and to Z_2 inside R_2. However, the random value is not clearly defined on grid cells that are partially on several domains. Let's consider the grid cell X of the Fig.2 which is not entirely inside any domain. To ensure the model continuity, we choose for this grid cell a combination of the two Gaussian white noises Z_1 and Z_2 as follows:

$$Z_X = a_1 Z_{1,X} + a_2 Z_{2,X} \tag{6}$$

where a_1 and a_2 depend on the shape and the size of the domains, which can be parameterized. As for the local gradual deformation method, Z is a standard Gaussian white noise if $a_1^2 + a_2^2 = 1$. We can choose, for example, a_i proportional to the proportion of the grid cell inside the domain i.

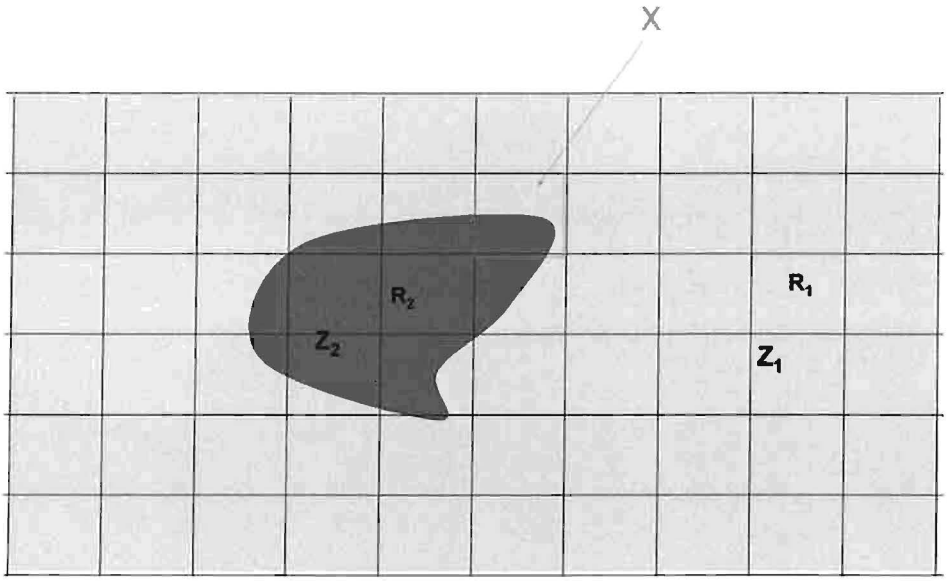


Figure 2: Reservoir model divided into 2 zones R_1 and R_2

The new model realization therefore depends on the parameters which define the shapes and sizes of the domains. In general, to limit the number of parameters, we choose simple shapes for the domains that depend only on a small number of parameters. For example, choosing circles with fixed centers allows us to determine each domain with only one parameter: their radius.

This method can be extended to the case of M domains to deform with $N+1$ Gaussian white noises. Let $t_i = (t^1, ..., t^q)$ be the set of parameters determining the shape of the domain R_i and $t = (t_1, ..., t_M)$ contain all the domain parameters, we can combine the Gaussian white noises:

$$Z(X,t) = \sum_{i=0}^{M} a_i(X,t_i) Z_{J(i)}(X) \tag{7}$$

where $a_i(X,t_i)$ depends on the shape of the domain R_i, $J(i) \in [0,\ldots,N]$ is the index of the Gaussian white noise associated to the domain R_i and $J(0) = 0$. The new standard Gaussian white noise is parameterized with t.

The advantage of this parameterization technique compared to the local gradual deformation is that it is not very dependent on the initial domain selections, since their shapes and sizes can be modified. In fact, a bad definition of the zones in the local gradual deformation method could greatly deteriorate the potential diminution of the objective function. This is well illustrated in Fig.3 which was presented by Ding & Roggero (2010) to compare the potential of the domain deformation method and the gradual deformation

method on a history matching problem. In this figure, the pink curve presents the variations of the objective function with the size of the domains, while the blue curve presents the optimal results using the local gradual deformation technique with fixed domains sizes. In this particular case, when the domain size (radius) is fixed, the contribution of the local gradual deformation is relatively limited while changing the domain size can rapidely reduce the objective function. This example shows that in certain cases, choosing well suited domains can be critical to the performance of the gradual deformation method.

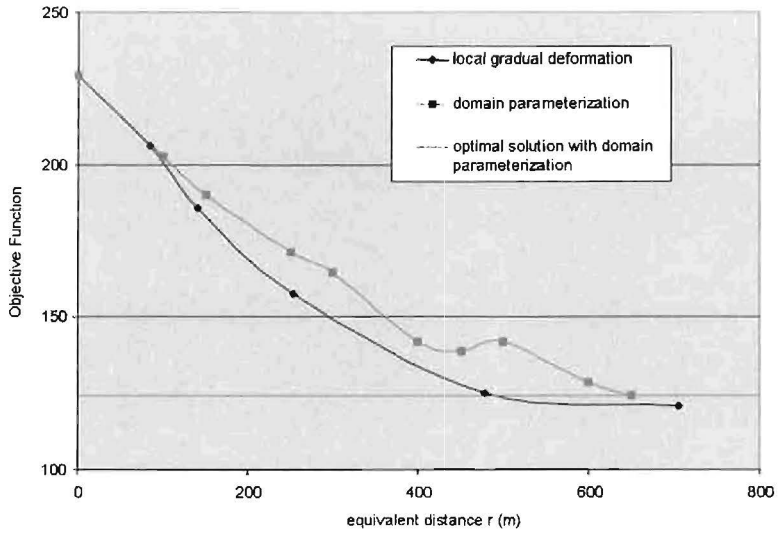


Figure 3: Comparison between the domain deformation method and the local gradual deformation method on a synthetic case

2.3. Generalization of the gradual deformation method

2.3.1. Another way to write the domain deformation technique

To be able to combine the gradual deformation and the domain deformation methods, we write them in a similar form. We present here a variant of the domain deformation method. Let's start by a simple case with only one zone and two model realizations.

One zone and two standard Gaussian white noises.

Let's Consider one zone R_1 parameterized by the set of parameters $t_1 = (t^1, ..., t^q)$ and two independent model realizations associated to two Gaussian white noises Z_0 and Z_1. For a given grid cell X, we define $V_1(X, t_1)$ by the volume of the domain R_1 inside X and $V_0(X, t_1) = vol(X - R_1)$ its complementary. Let α_1 be:

$$\alpha_1(X, t_1) = \frac{V_1(X, t_1)}{V_1(X, t_1) + V_0(X, t_1)} \tag{8}$$

A new gaussian white noise $Z(X)$ can be built with:

$$Z(X) = \cos(\frac{\pi}{2}\alpha_1)Z_0(X) + \sin(\frac{\pi}{2}\alpha_1)Z_1(X) \tag{9}$$

This Gaussian white noise associates Z_1 to the inside of the domain R_1 and Z_0 to the outside. It is a combination of the two on the grid cells partially occupied by the zone. The new model realization $G(Z)$ is dependent on the shape and size of R_1 which is parameterized with t_1.

Generalization: M zones and $M+1$ Gaussian white noises.

We can generalize the above formula to the case of M zones $(R_1, ..., R_M)$. If we have at our disposal $M+1$ Gaussian white noises $(Z_0, ..., Z_M)$ and a set $t = (t_1, ..., t_M)$ that parameterizes the shapes of all the zones, we can associate one model realization to each zone and the last one to the outside of every zone. As in the previous example, for a given grid cell X, we define $V_i(X, t_i) = vol(X \cap R_i)$, $V_0(X, t) = vol(X - \cup_j R_j)$ and α_i:

$$\alpha_i(X, t) = \frac{V_i(X, t_i)}{\sum_{j=0}^{M} V_j(X, t_j)} \tag{10}$$

For the sake of simplicity, we decided to associate for $i \in \{1, ..., M\}$ the Gaussian white noise Z_i to the zone R_i and Z_0 to the outside every zone. The new standard Gaussian white noise can be built using:

$$Z(X) = \prod_{j=1}^{M} \cos(\frac{\pi}{2}\alpha_j) Z_0(X) + \sum_{j=1}^{M} \sin(\frac{\pi}{2}\alpha_j) \left(\prod_{k=j+1}^{M} \cos(\frac{\pi}{2}\alpha_j) \right) Z_j(X) \tag{11}$$

We therefore have a model realization $G(Z)$ dependent on the shapes and sizes of the domains R_i built by associating one Gaussian white noise to one domain. With this formula, the domain deformation parameters are easier to manipulate. If a grid cell is entirely inside the zone R_i ($\alpha_i = 1$ and for all $j \neq i$, $\alpha_j = 0$) we have $Z(X) = Z_i(X)$.

2.3.2. Combination of the local gradual deformation and the domain deformation

Both the local gradual deformation method and domain deformation method previously described present some drawbacks. On one hand, it is not easy to define suitable zones for the local gradual deformation method to ensure a potential diminition of the objective function. On the other hand, the domain deformation does not allow the combination of Gaussian white noises inside the zones and therefore limits the possibilities of the optimization process. We propose in this section a new method that generalizes the local gradual deformation by allowing to deform the domains in which the model realizations are combined.

A simple example : One zone and two Gaussian white noises.

Let's start with a simple example where we have two Gaussian white noises (Z_0, Z_1) and one zone R_1 defined in a reservoir model. We propose a new Gaussian white noise that depends on two parameters with t_1 controling the domain R_1 and ρ^1 the gradual deformation.

$$Z(X) = \cos(\pi\alpha_1(X, t_1)\rho^1) Z_0(X) + \sin(\pi\alpha_1((X, t_1)\rho^1) Z_1(X) \tag{12}$$

where $\alpha_1(X, t_1)$ is given by Eq.(8).

Depending on the position of the grid cell X in the reservoir model and the value of the gradual deformation parameter, we have:

- If X is entirely outside R_1 then $\alpha_1 = 0$ and $Z(X) = Z_0(X)$.
- If X is entirely inside R_1 then $\alpha_1 = 1$ and

$$Z(X) = \cos(\pi\rho^1)Z_0 + \sin(\pi\rho^1)Z_1 \tag{13}$$

  which corresponds to the gradual deformation technique.

- If X is partially inside R_1, $Z(X)$ is a combination of $Z_0(X)$ and $Z_1(X)$. In this case, both the gradual deformation method and the domain deformation method are applied simultaneously.
- If the gradual deformation is set to a constant $\rho_1 = \frac{1}{2}$, then $Z(X) = \cos(\frac{\pi}{2}\alpha_1)Z_0(X) + \sin(\frac{\pi}{2}\alpha_1)Z_1(X)$, which corresponds to the domain deformation method described in section 2.3.1 (Eq.(9)).

One domain and $N+1$ Gaussian white noise.

We can extend the previous case by adding the possibility to combine $N+1$ independent standard Gaussian white noises $(Z_0, ..., Z_N)$ inside the zone R_1. t_1 still parameterizes the shape of R_1 and we introduce N gradual deformation parameters $(\rho^1, ..., \rho^N)$. The new Gaussian white noise proposed is then:

$$Z(X) = \prod_{j=1}^{N} \cos(\pi\alpha_1\rho^j)Z_0(X) + \sum_{j=1}^{N} \sin(\pi\alpha_1\rho^j) \left(\prod_{k=j+1}^{N} \cos(\pi\alpha_1\rho^k) \right) Z_j(X) \tag{14}$$

We have:

- If X is outside R_1, then $\alpha_1 = 0$ and $Z(X) = Z_0(X)$.
- If X is entirely inside R_1, then $\alpha_1 = 1$ and

$$Z(X) = \prod_{j=1}^{N} \cos(\pi\rho^j)Z_0(X) + \sum_{j=1}^{N} \sin(\pi 2\rho^j) \left(\prod_{k=j+1}^{N} \cos(\pi 2\rho^k) \right) Z_j(X) \tag{15}$$

  which corresponds to the formula of the gradual deformation method described in Eq.(3).

- If X is partially inside R_1, $Z(X)$ is a combination of $Z_0(X)$ and $Z_1(X)$ created by both the domain deformation and the gradual deformation techniques.

Extension to the case of $N+1$ model realizations and M domains.

We now extend the method to the general case where we have at our disposal $N+1$ independent Gaussian white noises $(Z_0, \ldots, Z_N)$ and M domains $(R_1, \ldots, R_M)$ defined on a reservoir model. For each zone R_i, let α_i be defined by Eq.(10) and for each couple "zone"/"Gaussian white noise" R_i/Z_j, we introduce a gradual deformation parameter ρ_i^j. We propose the generalized formula :

$$Z(X) = \prod_{j=1}^{N} \cos(\pi \sum_{i=1}^{M} \alpha_i \rho_i^j) Z_0(X) + \sum_{j=1}^{N} \sin(\pi \sum_{i=1}^{M} \alpha_i \rho_i^j) \prod_{k=j+1}^{N} \cos(\pi \sum_{i=1}^{M} \alpha_i \rho_i^k) Z_j(X) \tag{16}$$

This is indeed a way to do a gradual deformation of all model realizations in every varying zone but this generalized method introduces too many parameters ($M * (N+1)$). Fortunately, it is possible to reduce the number of parameters just by selecting the "best" model realizations to combine in each zone (setting some of the ρ_i^j to 0).

Let's analyse some cases for a grid cell X:

1. Outside every domains

   For each i, $\alpha_i = 0$. Thus, $Z(X) = Z_0(X)$.

2. Suppression of the influence of the realization l in the zone R_q

   Let's set the parameter ρ_q^l to 0. If the grid cell X is entirely and uniquely inside R_q, $Z(X)$ become:

   $$Z(X) = \prod_{j=1, j\neq l}^{N} \cos(\pi \rho_q^j) Z_0(X) + \sum_{j=1, j\neq l}^{N} \sin(\pi \rho_q^j) \prod_{k=j+1}^{N} \cos(\pi \rho_q^j) Z_j(X) \tag{17}$$

   The realization l has no more influence in the zone R_q.

3. Domain deformation of the zone q with only the realization l associated to R_q

   We want to apply Z_l inside R_q. Let's set $\rho_q^l = \frac{1}{2}$ and for all $j \in \{1, \ldots, l-1, l+1, \ldots, N\}$ the parameters $\rho_q^j = 0$. Then:

   - If X is entirely and uniquely inside R_q then $Z(X) = Z_l(X)$
   - If X is only partially inside R_q, then a combination of Z_0 and Z_l is applied, as for the domain deformation method.

   This case corresponds to the domain deformation method applied on the zone R_q.

4. Gradual deformation on the zone R_q

   If no parameter controlling the shape of R_q is introduced then for any grid cell X, α_q is constant. This ammounts to perform a local gradual deformation inside the zone R_q.

This new generalized method offers a great flexibility concerning the definition of different domains and the choice of Gaussian white noises to combine in each one. For example, it is possible to perform a local gradual deformation of several model realizations in one zone, while in parralel applying the domain deformation method to another zone (or any combination of the two methods).

3. Initialization of the history matching problem : a patchwork technique

One of the issues for the history matching problem is its initialization. For the geostatistical local parameterization techniques described in the previous section, choosing relevant domains and model realizations is a key issue to have a good decrease of the objective function. It is therefore necessary to have an efficient way to initialize the history matching problem.

3.1. Initialization of the history matching problem via patchwork

By dividing a reservoir model into relatively independent zones, it is possible to initialize the history matching problem by choosing the "best" realizations in each of these zones. The choice of model realizations for a zone can be done by comparing the local value of the objective function and selecting the best one. Let's present the patchwork technique (Reis et al., 2000).

The objective function can be explicited by:

$$F = \sum_{i=0}^{n} f_i(x_1, \ldots, x_p) \tag{18}$$

where f_i is the objective function associated to a well i on which we have data to match, n is the number of wells in the reservoir and $(x_1, \ldots, x_p)$ are the parameters to be optimized. For a specific zone k, if I is the set of wells inside the zone k, we call local objective function associated to the zone k the function $g_k = \sum_{i \in I} f_i$.

A patchwork initialization is performed by the following steps:

1. generate $N + 1$ model realizations.
2. divide the reservoir into M zones.
3. Name Z_0 the Gaussian white noise associated to the model realization that presents the lowest objective function value.
4. For each zone k:
   - Sort the values of the local objective function. Name Z_j the realization such that $g_k(Z_j)$ is minimal.
   - If $Z_j \neq Z_0$, apply Z_j in the zone k for gradual deformation.

Fig.4 shows an example of a reservoir model divided into 11 relatively independent zones. If the zones are independent from one another, the patchwork initialization is expected to give a better objective function value than Z_0. However, if the local modification of the model realization in a zone can affect the behavior of the model in the other zones, it is not possible to ensure that the patchwork method will not deteriorate the model. In practice, the independence of different regions is not trivial to be identified and large errors could be committed. It is frequent that a patchwork method increases the value of the objective function.

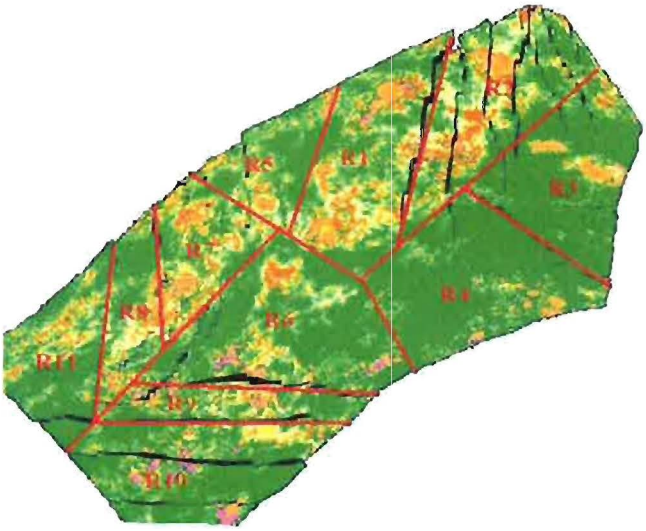


Figure 4: Separation of a reservoir model in 11 independent zones

3.2. Improved patchwork

With the new parameterization technique or the domain deformation method, it is more flexible to initialize local zones since their shapes and sizes can be later modified. The patchwork method is improved as follows:

1. Generate $N + 1$ model realizations.
2. Name Z_0 the Gaussian white noise associated to the model realization that presents the lowest objective function value.
3. Group the wells into M subsets $(I_1, ..., I_M)$. As in the previous section, we call $g_k = \sum_{i \in I_k} f_i$ the local objective function associated to the k^{th} group of wells. It is possible to choose only one well in each group.
4. For each group of wells k:
   - Sort the values of the objective subfunction $g_k(Z_j)$. Name Z_j the realization such that $g_k(Z_j)$ is minimal.
   - If $Z_j \neq Z_0$, define a parametrized zone containing the group of wells k and apply Z_j inside. Simple zones defined with one or two parameters are prefered.
5. Start a simple optimization process (for instance, with a single parameter) on the sizes of the domains to have the best possible starting model. In the case of circular zones, we can optimize the sizes of the zones with a single radius parameter.

Comparing to the patchwork proposed by Reis et al. (2000), parameterized domains are defined in Step 4 and any form of domains can be used instead of a division by regrouping grid cells. Moreover, a simple optimization process is applied in Step 5 to improve the zone sizes. As we will see in examples hereafter, Step 5 allows with a very limited cost (a few reservoir simulations) to get a better initial point.

It is interesting to note that our method leaves a lot of freedom concerning the shape of the zones. Ding & Roggero (2010) presented several typical domains for history matching. Elliptical domains are suitable for anisotropic permeability correlation media with their two axes proportional to the correlation lengths. They are also adapted to investigate connectivities between a couple of injector/producer with the foci of the ellipse at the wells. The simplest domain shape is the circle with its center fixed at a well and a radius r to be optimized. In the following, we will illustrate some examples using radial domains parameterized with their radius.

4. Numerical results

In this section, we tested our new initialization and parameterization techniques on three cases. The first one is a simple synthetic reservoir model, the second one is the reservoir model PUNQ (Barker et al., 2000) and the last one is an adapted Brugge model (Chen et al., 2010, Peters et al., 2010, 2012). In the simple synthetic case and the Brugge case, as the correlation lengths are nearly isotropic, we worked with circular zones. Anisotropic correlation lengths are present in the PUNQ reservoir, but each layer has a different anisotropy and different axis: their azimuths are respectively 30, 0, 45, -30 and 60 degrees from the principal axis. As it is difficult to favor a particular layer over the others and we don't want to introduce one new parameter per layer around each well, we also chose to work with circular domains for this test. We will briefly describe the studied cases before showing the numerical results.

4.1. Description of the three studied cases

Simple synthetic case. The dimensions of the field are 2500m in the x direction, 2500m in the y direction and 10m in the z direction. It is discretized uniformly by 50x50x1 grid cells of 50m in the x direction, 50m in the y direction and 10m in the z direction. There are 25 vertical wells containing 12 water injectors and 13 producers (Fig.5). The reservoir is heterogeous and the correlation length is 150m in the x and y directions. The initial reservoir pressure is 250bars and a production data history of 4000 days is known. Well bottom hole pressures are imposed with 320 bars on the injectors and 180 bars on the producers. We consider for our objective function to be minimized the water rate on the 12 injectors as well as the oil rate and the water-cut on the 13 producers.

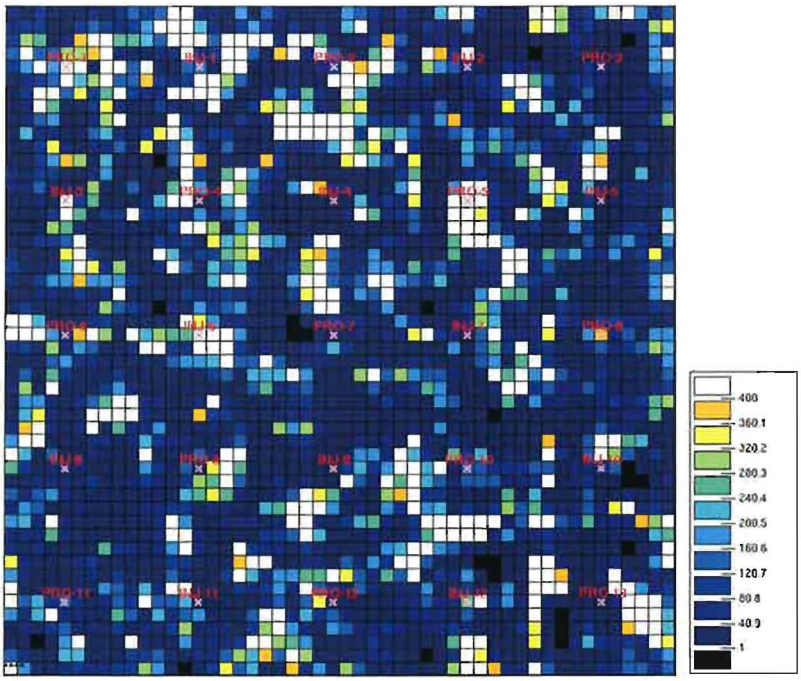


Figure 5: Permeability map of the synthetic oil field

PUNQ. The reservoir PUNQ is a synthetic case which results from a field study conducted by the industrial partners of the project PUNQ (Barker et al., 2000). The model is constituted of 19x28x5 grid cells of which 1761 are active. Six producing and seven injection wells are present. Fig.6 shows the permeabilty at the top layer of the reservoir for a realization of the model. Porosity and permeability maps have been generated with the geostatistical method FFTMA (Le Ravalec et al., 2000). We have at our disposal a production history of 12 years. We use for our objective function the parameters BHFP (Bottom Hole Flowing Pressure), WC (Water CUT) and GOR (Gas Oil Ratio) for the producing wells and BHFP on the injecting wells.

Brugge. Brugge is a synthetic reservoir model built for the 2008 Applied Technology Workshop (Peters et al., 2010, 2012). The original model has 20 millions grid cells with average sizes of 50m x 50m x 0.25m. We worked with an upscaled model of 60 thousands grid cells. 30 wells are present on the reservoir, 10 are injecting wells and 20 are producing wells. In this work, we generated our history data by creating reference porosity and permeability maps with the geostatistical method FFTMA (Le Ravalec et al., 2000), and we compare different optimization techniques to match these history data. Figure 7 shows the permeability map of the first layer of the reservoir for the reference realization of the model. The production has a history of 10 years. We use for our objective function the parameters BHFP (Bottom Hole Flowing Pressure) and WC (Water Cut) for the producing wells and BHFP on the injecting wells.

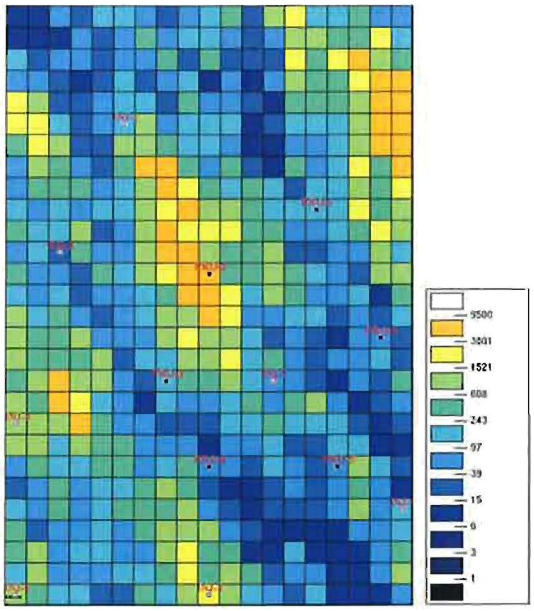


Figure 6: Permeability and well position of the top layer of the PUNQ reservoir

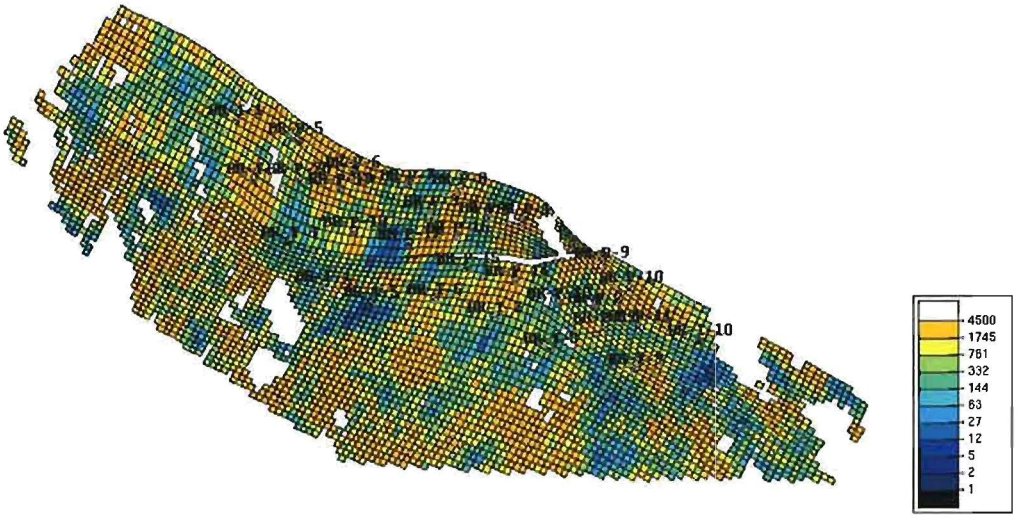


Figure 7: Permeabilty and well positions on the Brugge reservoir

4.2. Model initialization

Simple synthetic case. We generated 5 sets of 25 random realizations to apply the patchwork method with radial domains around the wells. Fig.8 illustrates the new patchwork method: the zones created are the circles around the wells. In this figure 16 zones were created, the problem is therefore parameterized with 16 gradual deformation parameter and 16 domain deformation parameters (radius of the domains).

Fig.9 shows the values of the objective function in each of our 5 cases for the best realization and the

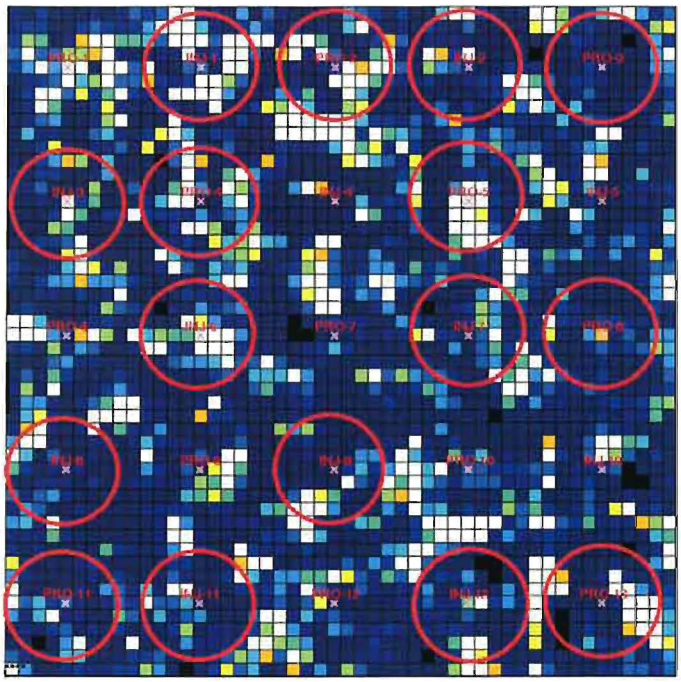


Figure 8: Initialization for the new parameterization method via modified patchwork on a synthetic reservoir model

realization given by the patchwork technique with fixed a radius $r = 300m$ on all the domains. For the first 4 cases, the patchwork method is beneficial without any further evaluation of the objective function. However, for the last case, the objective function has been increased by the patchwork technique. In consequence, we

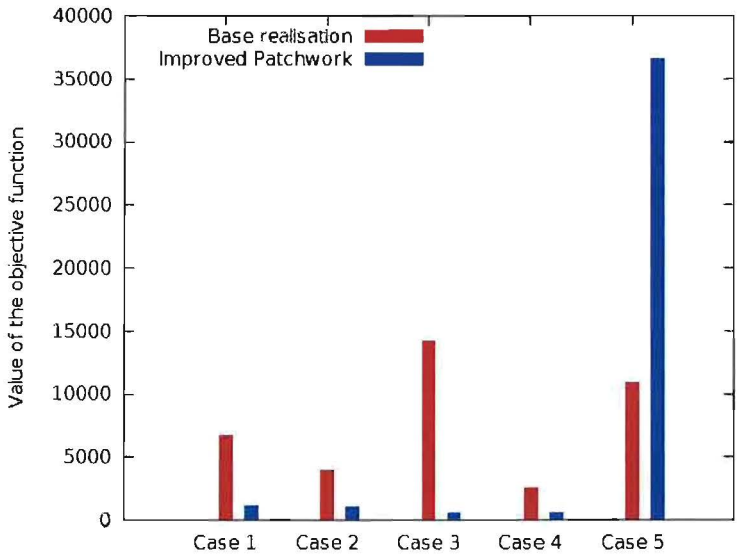


Figure 9: Value of the objective function for the best realization (in red) and for the realization obtained by patchwork with $r = 300m$(in blue) in 5 cases

drew for that case in Fig.10 the value of the objective function after patchwork as a function of the domains radius. It is observed that the objective function is deteriorated with the radius $r = 300m$. However, there is a clear minimum on this curve that is inferior to the value of the "best" realization (horizontal bar in blue). A gradient based optimization method can find this minimum in 5 evaluations of the objective function. Radial domains give a better initial model with a radius of $r = 206$m for the optimization proccess.

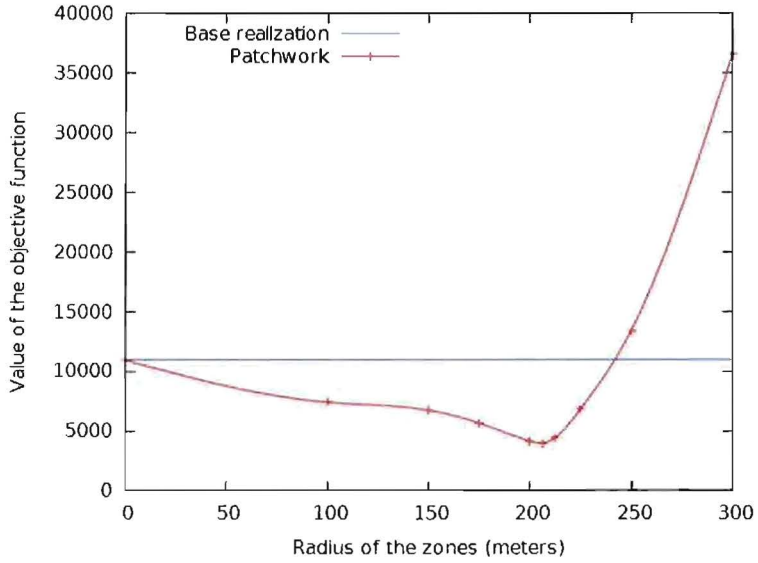


Figure 10: Objective function after patchwork in respect with the radius of the zones

PUNQ. As for the previous example, we generated 11 random model realizations for the PUNQ case. The new patchwork technique created 9 effective zones because the local objective functions associated to the four other wells are the minimum on the base case (the "best" random realization). Fig.11 shows the value of the objective function after patchwork with respect to the radius of the zones. The minimum is clearly inferior to the value of the objective function given by the "best" realization. We found this minimum with 7 evaluations of the objective function through a gradient-based optimization method. The objective function goes from 649.9 to 502.0 with the new patchwork approach showing a relative gain of 22%.

To compare with the initial point given by the classical patchwork technique (Reis et al., 2000), we divided the grid into zones delimited by grid cells as shown in Fig.12. In this example, each zone contains one and only one well. Using this domain division, the objective function increases from 649.9 for the best global realization to 851.1. Unfortunately, the classical patchwork method is detrimental to the objective function but we have no mean to improve it.

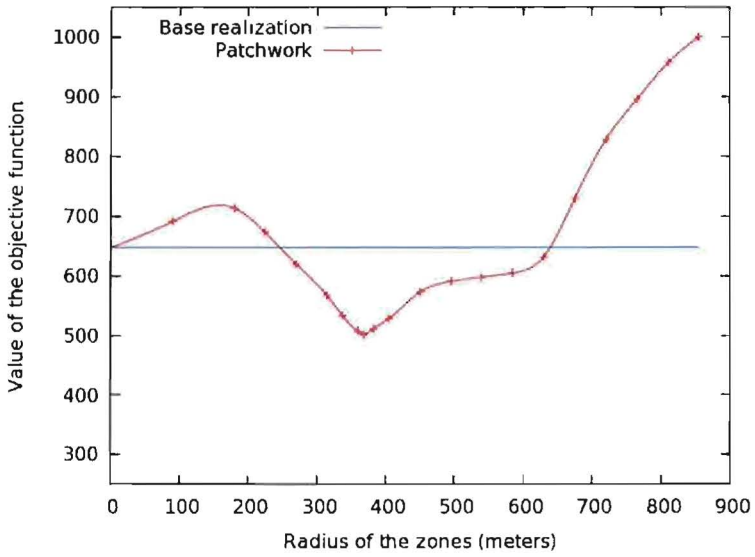


Figure 11: Value of the objective function after patchwork in respect with the radius of the zones for the PUNQ case

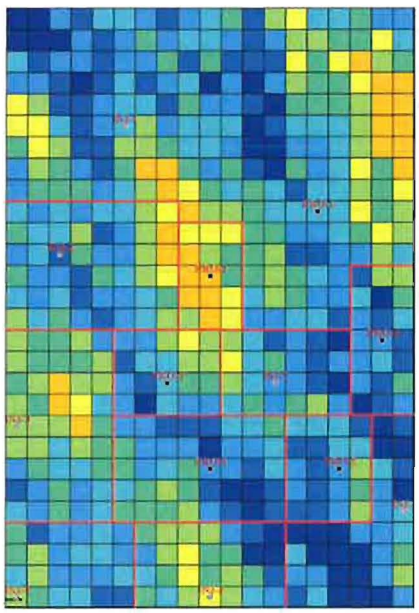


Figure 12: An engineering division of the PUNQ reservoir into zones

It is interesting to note that even in the case where the objective function cannot be improved, the new patchwork method is not detrimental. In fact, when all the radius are reduced to zero, we obtain the "best" realization among all the random realizations.

Brugge. For this case, 19 random model realizations were generated. The new patchwork technique created 7 zones around the wells $P-10$, $P-13$, $P-15$, $P-16$, $P-19$, $P-2$ and $P-5$. Fig.13 shows the value

of the objective function after patchwork with respect to the radius of the zones. Again, the minimum is inferior to the value of the objective function given by the best realization. We could find this minimum in

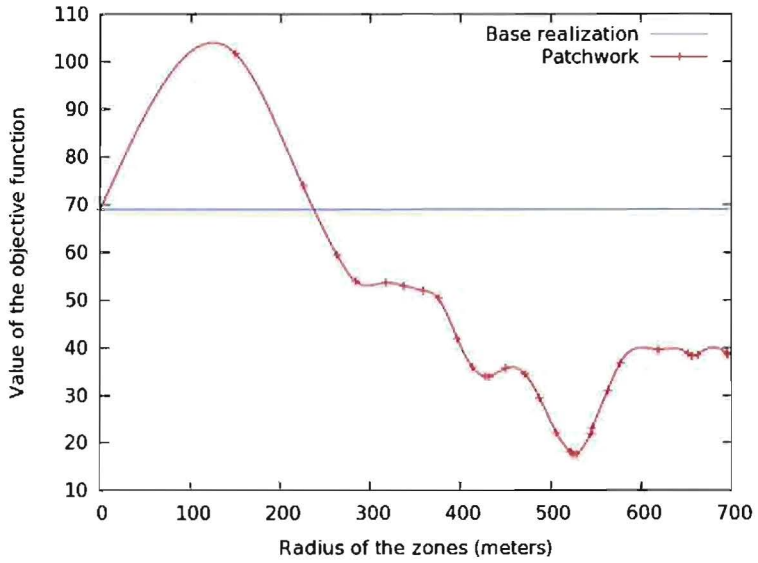


Figure 13: Value of the objective function after patchwork in respect with the radius of the zones for the Brugge case

only 6 objective function evaluations using a gradient based optimization method for one radius parameter. The objective function goes from 68.9 to 17.5 with the patchwork approach, showing a relative gain of over 74%.

As for the PUNQ case, we compared our results to the one given by the classical patchwork method where we created zones delimited by grid cells around the wells (Fig.14). Using this reservoir division, the patchwork technique gives an objective function value of 45.2. In this case, the method improves the objective function but is still far from the results given by our method.

4.3. Numerical results of the generalized gradual deformation method

We use the model obtained by patchwork in the previous session as the starting point for the history matching. In all tests, a gradual deformation parameter is introduced on each zone to combine the local best realization with the overall best random realization, and a domain parmeter is also introduced to control the radius of the zones.

Simple synthetic case. The generalized gradual deformation method is compared to the local gradual deformation. A gradient based optimization approach has been applied in the optimization process. Fig.15 shows the decrease of the objective function with respect to the number of evaluations for the five cases.

In each case, the decrease rate of the objective function with respect to the number of reservoir simulations obtained with the generalized gradual deformation method is similar to the one obtained with the classical

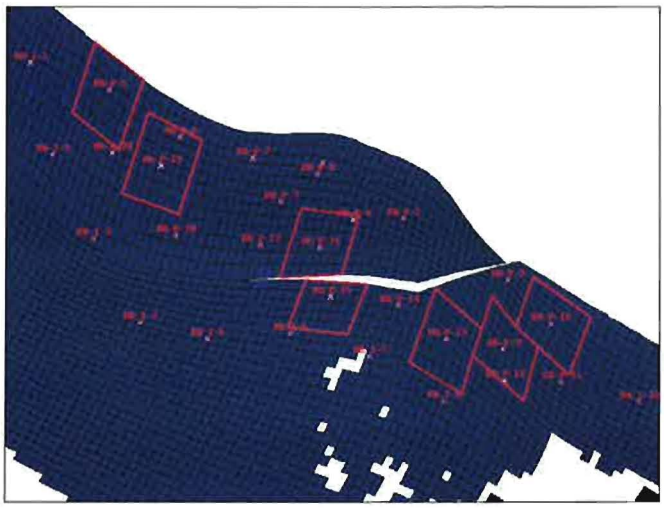


Figure 14: An engineering division of the Brugge reservoir into zones

local gradual deformation method, but the generalized gradual deformation method gives a better optimal result at the end (respectively 5%, 11% , 20%, 47% and 27% relative gain on the gradual deformation method). Even with an increased number of parameters, this new parameterization method gives a better result in a similar number of objective function evaluations. These results show the importance of taking into account the size of the local zones for history matching for the local gradual deformation.

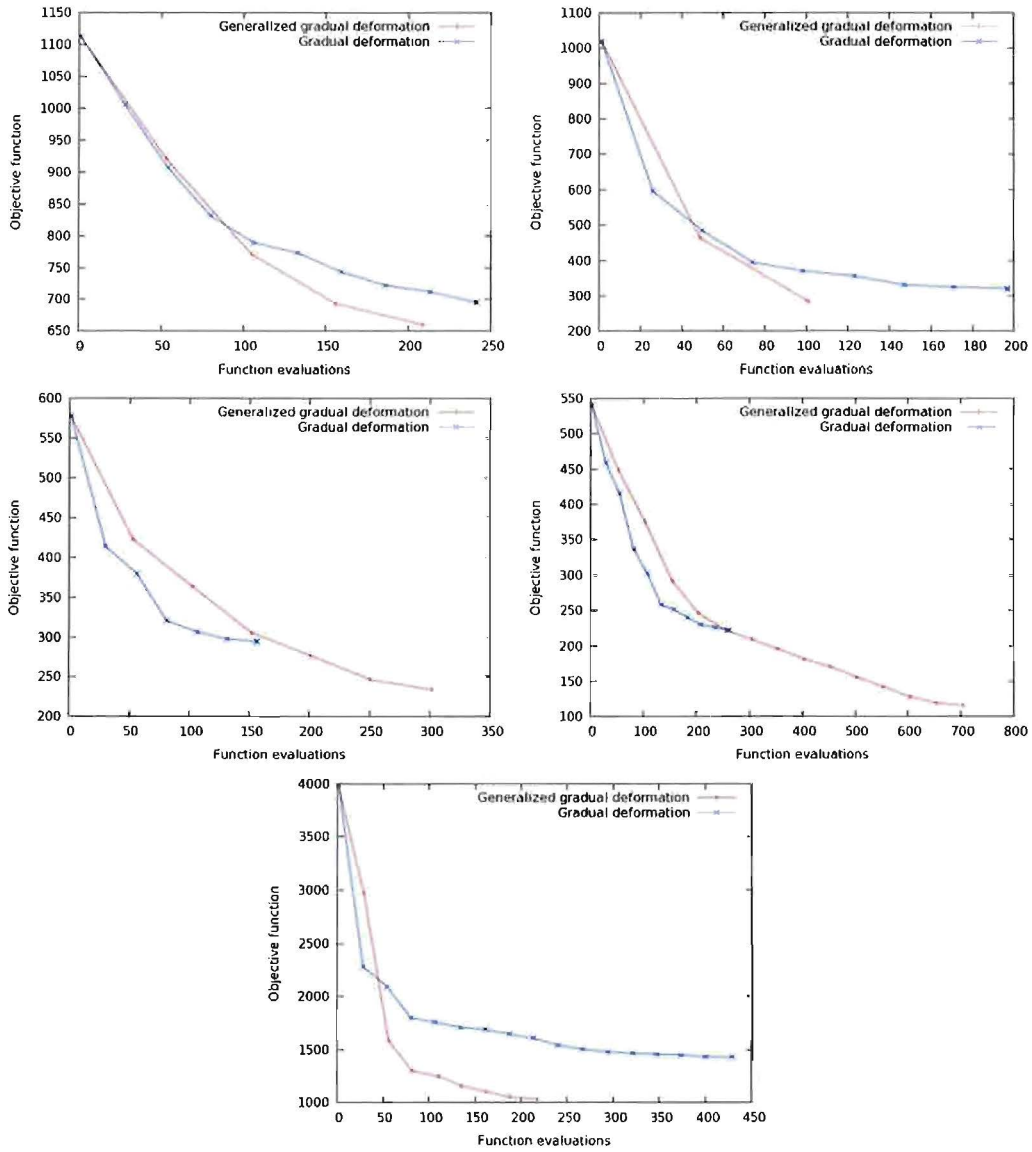


Figure 15: Comparison of the generalized gradual deformation and the gradual deformation techniques on 5 instances of the synthetic case.

PUNQ. We work in this test with 9 zones (see section 4.2). We start by comparing the performances of our generalized method and the local gradual deformation method on an engineering defined zones delimited by grid cells (Fig.12). Only the definition of the zones differs between the two methods : we consider the same number of zones and combine the same model realizations inside the zones. Results are shown on Fig.16.

The optimization starts with the base realization (best realization among the 10 random ones initially generated). The first step on the generalized gradual deformation curve corresponds to the initialization technique (section 3) and illustrates that we needed 7 evaluations to find a better initial point by optimizing

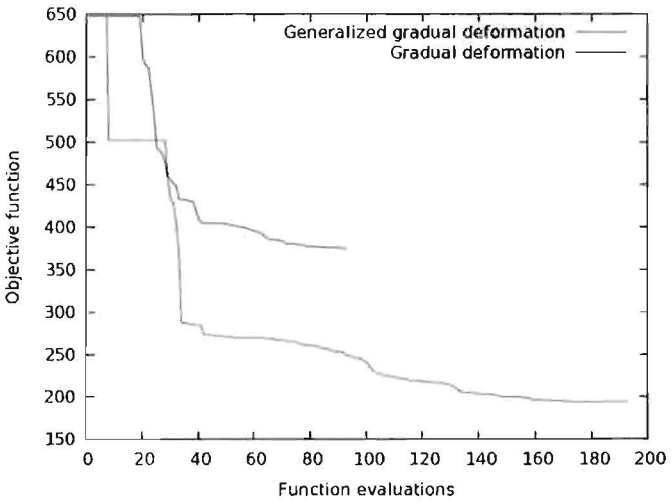


Figure 16: Optimization of the objective function using the local gradual deformation and the generalized gradual deformation on the PUNQ case

the size of the zones. The generalized gradual deformation method gives a far better result than the classical gradual deformation technique (relative gain of over 48%).

To further illustrate our method, we also compared three variant parameterization techniques from the best initial point obtained with the improved patchwork: the generalized gradual deformation method (9 radius parameters as we have 9 zones), the generalized gradual deformation method with one radius parameter for all the zones and the local gradual deformation method where the radius are unchanged. For this case, a derivative free optimizer (Langouët, 2012) was chosen to minimize our objective function. Results are presented in Fig17.

We can see in this figure that all the three methods give a similar decrease rate for the objective function. However, the methods that allow the sizes of the zones to vary give better final results (about 15% relative gain). Moreover, it seems sufficient to introduce only one parameter to controle the sizes of all the zones in this example. These simulations also show that taking into account the sizes of the zones can add a great flexibility to the parameterization technique without increasing greatly the cost of the history matching process.

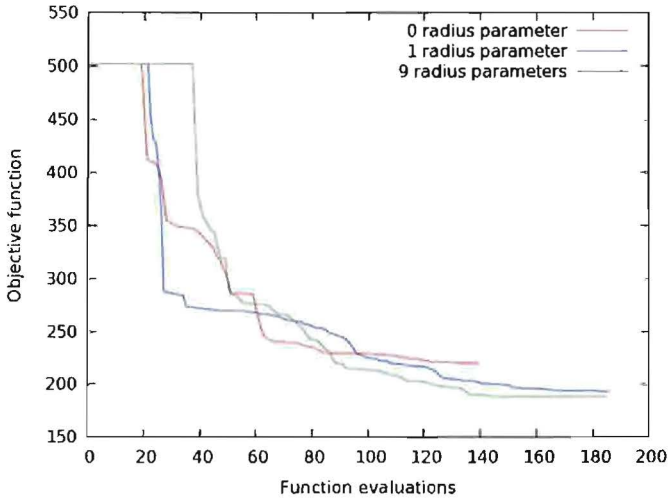


Figure 17: Optimization of the objective function for the 3 parameterization techniques on the PUNQ case

Brugge. We work in this test with 7 zones (see section 4.2). We performed two history matching, one using the local gradual deformation method on the zones delimited by grid cells (Fig.14) and one using our parameterization method. We show in Fig.18 the results we obtained.

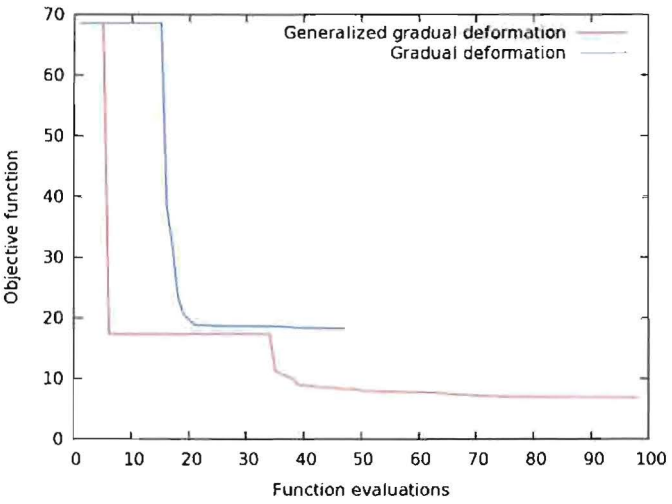


Figure 18: Optimization of the objective function using the local gradual deformation and the generalized gradual deformation on the Brugge case

In this case, the initialization method alone gives a better model realization than the local gradual deformation method. Our method presents a final relative gain of 60%. It is quite evident that a good definition of the zones is crucial to have a fast decrease of the objective function for this test.

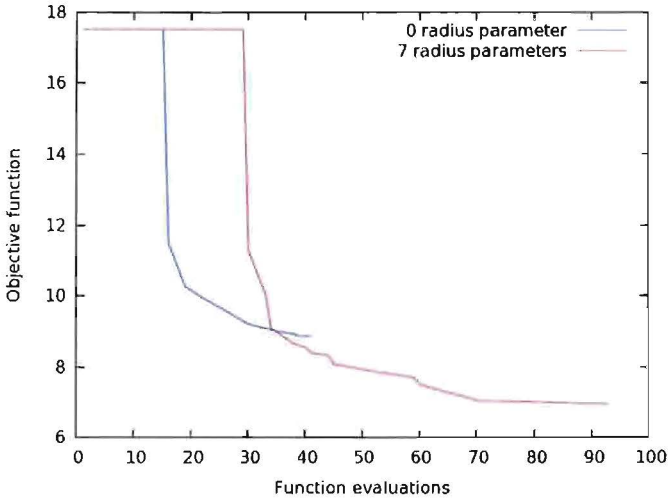


Figure 19: Optimization of the objective function for two variant parameterization techniques on the Brugge case

We also compared two variant parameterization techniques, one with 7 radius parameters and one without radius, from the best point obtained with the improved patchwork. We optimized the history matching objective function using the SQA (Langouët, 2012) optimizer. Results are presented on Fig 19. As in the previous tests, we were able to get a better model realization with a radius parameter for each zone (relative gain of 21%). Taking the size of the zones into account allows for a greater decrease of the objective function.

5. Conclusion

We presented in this paper a new parameterization method that remedies one of the biggest flaws of the local gradual deformation technique in history matching. The quality of the local gradual deformation method depends strongly on the initial set of local domains. To overcome this shortcoming, a generalized formulation is proposed, which modifies the geostatistical realization with the gradual deformation method inside local domains while the shapes and sizes of these domains are also changed by the domain deformation technique. This method allows the history matching optimization algorithm to dynamically modify the sizes of the domains and therefore reduces the dependence of the optimal realization on an initial expert guess for local domains selections. We successfully showed better results with our generalized gradual deformation method than those with the standard gradual deformation technique on three history matching problems. Therefore we are confident that this new method is a good generalization of the gradual deformation technique and may allow good history matching on a greater number of cases.

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